Collaborative Insurance Sustainability & Network Structure

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INTRODUCTION

The peer-to-peer (P2P) economy is characterized by its decentralization - individuals interact directly with each other rather than via a third party. Instead of needing to be used to expedite the connection between individuals, where Uber or Airbnb are prominent examples of such facilitators. Recent advances in the actuarial literature demonstrate how such a P2P approach based on networks might be incorporated into the insurance industry. Whereas previous research such as Feng et al. (2021) or Denut and Robert (2020) studied connections on homogenous networks with heterogenous risks, in this article (Charpentier et al., 2021) we instead consider the sustainability of such a system by considering different structures of networks themselves and assume homogenous risks due to homophily. We then develop a mechanism that supports a range of network structures.

RISK SHARING WITH FRIENDS

Here we consider the simple case where risk is exchanged on a network via the nodes in form of reciprocal contracts between agents. In the case presented here, we assume that all \( n = 1000 \) agents, with average degree \( \bar{d} = 20 \), who are exposed to the same risk that either materializes into a claim or not within a given period according to an i.i.d. Bernoulli variable \( Z_i \) with \( p = 0.1 \). If a claim is recorded, a cost \( Y_i \sim \Gamma(100, 2000) \) is associated to it. Further, we consider the case where the total cost is capped above at \( z = 10000 \) (any amount above that might be covered by a traditional insurer). This setup will result in an expected loss of around 1.5% of the deductible and a loss standard deviation of around 17% of the deductible. A desirable risk sharing mechanism should result in a lower standard deviation.

Identical contracts across the network

The simplest case is to include identical contracts for every edge in the network. A natural bound on the amount would be \( \gamma = \frac{d}{2} = 50 \), such that each agent could cover their entire deductible in a regular network. This proper risk-sharing mechanism will then assign the following cost to every agent \( i \) with connections \( V_i \) at the end of each period:

\[
\xi_i := \min\{x_i, Y_i\} + \sum_{j \in V_i} Z_j \min\{\min\{x_j, Y_j\}, d_{ij}\} - Z_i \min\{d_i, \min\{x_i, Y_i\}\}
\]

Where the gray part corresponds to the non-insurance case and the two terms on the right correspond to the debits and credits to \( i \) due to the transfers. Results from simulations demonstrate that the simple mechanism works well for rather regular networks, but it results in outcomes even worse than the no insurance case for high-variance networks. This is due to the concentration of some nodes that will assume more than their optimal share of contracts.

Formulation as LP

Instead of having fixed amount contracts, we can introduce the amounts into an optimization problem. We formulate a linear program that maximizes total coverage within the network but subject to the constraint that every agent only wants to cover at most \( z \). In the high variance case this will naturally lead to some contracts having \( \gamma = 0 \). We further introduce some self-contribution to alleviate the issues from above. Results of the simulations are depicted to the left with indicators for the different levels of self-contribution.

EXTENSIONS TO THE SIMPLE MODEL

The mechanism, even with optimized reciprocal contracts, still struggle to perform well in the high-variance cases. Further, the assumption of i.i.d. risks across the whole network might seem overly simplistic. Here we propose two extensions to mitigate these issues.

Higher order connections

Instead of only considering sharing risk with friends, the framework can be extended to include friends-of-friends. This just corresponds to the connections of the second degree, which can easily be calculated using matrix algebra. We assume trust between parties is smaller in the second degree and hence the agents will want to share less of their risk with the second degree connections. In the example to the left we assumed that \( \gamma_1 \), the amount shared with first order connections is equal to 25 and \( \gamma_2 = 5 \). The amount sharable by first degree connections decreases with higher degree variance but the amount sharable by second degree connections actually increases (and enables more agents to fully insure their entire risk). This arises because high variance networks generate hubs that act as pass-through points similar to the central nodes in star shaped networks.

Risk sharing on subnetworks (heterogeneous risks)

Instead of assuming homophily on the whole network, we can also pose the assumption on a smaller level. An example for such a case is a peer-to-peer household insurance within a city. Consider for example theft insurance, usually theft occurrences will be homogenous within a community but less so across. Such connections can again be modelled by a network. Instead of directly optimizing the linear program as before, we can employ an algorithm to find more densely connected subgroups and impose additional constraints on the LP. This in turn should alleviate issues concerning the sustainability in heterogeneous risk cases. One measure of success could be the total amount shared within each true subgroup as depicted above in (b), or in future work, dynamics of such solutions could be analysed.

REFERENCES


Graph Representation

Our goal is to derive a risk sharing mechanism based on variety of different networks of agents. To cover the general case, we express a network as a graph \( G = (V, E) \), where \( V \) is the set of vertices or nodes that will represent agents that would like insure themselves against a risk and \( E \) is the set of edges or links that represent ties between the agents. We will assume undirected edges, \( i \leftrightarrow j \) meaning that if \( i \) has ties to \( j \), \( j \) has ties to \( i \). An important measure to quantify how regular a graph is its degree vector \( d \) which contains the number of adjacent vertices for every node in the graph \( G \) (that is \( d = |\{j: (i, j) \in E\}| \)). Simply changing the dispersion of the degree vector results in a array of different networks. On one extreme we have low-variance regular graphs such as encountered in Feng et al. (2021) and in the high-variance case almost star shaped networks such as they are often encountered in social media. Ensuring the sustainability of an insurance mechanism is highly dependent on the shape of a network. Even in the case of homogenous risk within a network, when risk is to be shared via edges, only regular networks guarantee a stable number of sharing possibilities for each agent. In the high-variance case, as depicted right subfigure (b), some agents only have a single connection and efficient risk sharing via edges becomes more difficult. As that follows we simulate different networks with the degree distribution simulated as:

\[
D = \min\{5 + |\Delta|, n\}, \quad \Delta \sim \Gamma(\bar{d} - 5, 5^2)
\]