Actuarial Pricing Game (Season 2)

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http://freakonometrics.hypotheses.org
A Game with Rules... but no Goal

Warning 1
For participants: forget about individuals results, focus on the global picture

Warning 2
Difficult to pool the results together (some mistakes are probably mine...)

Warning 3
The ‘pricing game’ has rules.
But no goal was ever mentioned.
Every player played according to his/her goals.
**Context (and Apologies)**

Two datasets: a *training* one, and a *pricing* one (without the losses in the later)

**Step 1**: provide premiums to all contracts in the pricing dataset

**Step 2**: (somme) premiums provided by competitors were given to players

**Warning 4**

Motor insurance, but on fleats: one ‘contract id.’, several vehicles

**Warning 5**

Technical issue: the exposure was still in the *pricing* dataset

**Warning 6**

!normalization: \( \sum_{i=1}^{n} \pi^{(j,1)}(x_i) = \sum_{i=1}^{n} \ell_i = 2.5M \) and \( \sum_{i=1}^{n} \pi^{(j,2)}(x_i) = 2.35M, \forall j \)

loss ratio = 100%
Part 1: Standard Pricing Game, a Global Overview

Consider an ordered sample \( \{y_1, \cdots, y_n\} \) of incomes, with \( y_1 \leq y_2 \leq \cdots \leq y_n \), then Lorenz curve is

\[
\{F_i, L_i\} \text{ with } F_i = \frac{i}{n} \text{ and } L_i = \frac{\sum_{j=1}^{i} y_j}{\sum_{j=1}^{n} y_j}
\]

We have observed losses \( \ell_i \) and premiums \( \hat{\pi}(x_i) \). Consider an ordered sample by the model, see Frees, Meyers & Cummins (2014), \( \hat{\pi}(x_1) \geq \hat{\pi}(x_2) \geq \cdots \geq \hat{\pi}(x_n) \), then plot

\[
\{F_i, L_i\} \text{ with } F_i = \frac{i}{n} \text{ and } L_i = \frac{\sum_{j=1}^{i} \ell_j}{\sum_{j=1}^{n} \ell_j}
\]
Part 1: Standard Pricing Game, a Global Overview

Dispersion of the premium, per company, $\pi^{(1,j)}(x_i), j \in \{1, 2, \cdots, 14, 14^*\}$. 
Market Competition

**Decision Rule:** the insured selects randomly from the three cheapeast premium

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<th>Ins3</th>
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<th>Ins5</th>
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Part 1: Standard Pricing Game

Market shares of the 12 insurance companies
Part 1: Standard Pricing Game

Premium variability (overall) and loss ratios of the 12 companies
Part 2: Pricing Game with Additional Information
Part 2: Pricing Game with Additional Information

New Lorentz curves:
order policies according to the premium

$$\pi^{(2,j)}(x_1) \geq \pi^{(2,j)}(x_2) \geq \cdots \geq \pi^{(2,j)}(x_n)$$

then plot

$$\{F_i, L_i\} \text{ with } F_i = \frac{i}{n} \text{ and } L_i = \frac{\sum_{j=1}^{i} \ell_j}{\sum_{j=1}^{n} \ell_j}$$
Part 2: Pricing Game with Additional Information

Premium variability (overall) and loss ratios of the 12 companies
Part 2: Pricing Game with Additional Information

Premium variability (overall) and loss ratios of the 11 companies
Part 2: Pricing Game with Additional Information

Premium variability (overall) and loss ratios of the 12 and the 11 companies
Smaller Market Competition (3 companies) : Step 1
Smaller Market Competition (3 companies) : Step 2
Smaller Market Competition (3 companies)

**Decision Rule:** out of 3 companies the insured selects (i) the **cheapest premium** (ii) randomly from the **two cheapeast premiums**, with probabilities 2/3 (cheapest) and 1/3 (second cheapest)

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Smaller Market Competition (3 companies)

Step 1: market shares and loss ratios
Smaller Market Competition (3 companies)

Step 2: market shares and loss ratios ((i) and (ii))
Smaller Market Competition (3 companies)

Loss ratios, Step 2 vs. Step 1 ((i) and (ii))
Behaviors of Insurance, Local Perspective: Ins4

Insurer 4 has informations about premiums provided by Insurer 5 and Insurer 6 (after Step 1). Only 50% of the premiums were observed.

Behaviors of Insurance, Local Perspective: Ins5

Insurer 5 has informations about premiums provided by Insurer 4 and Insurer 6 (after Step 1).
Only 50% of the premiums were observed.

Ins 4. vs. Ins 5.  
Ins 6. vs. Ins 5.  
Step 2 vs. Step 1.
Behaviors of Insurance, Local Perspective: Ins6

Insurer 6 has informations about premiums provided by Insurer 4 and Insurer 5 (after Step 1). Only 50% of the premiums were observed.

Ins 4. vs. Ins 6.  
Ins 5. vs. Ins 6.  
Step 2 vs. Step 1.
Behaviors of Insurance, Local Perspective

Very different behaviors, for various insurance companies

Ins. 2

Ins. 13

Ins. 14