Equilibria for Insurance Covers of Natural Catastrophes on Heterogeneous Regions

Arthur Charpentier (Université de Rennes 1, Chaire ACTINFO)

& Benoît le Maux, Arnaud Goussebaïle, Alexis Louaas

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http://freakonometrics.hypotheses.org
Proportion of insurance policy that did claim a loss after storms, for a large insurance company in France (~1.2 million household policies)
**Demand for Insurance**

An agent purchases insurance if

$$\mathbb{E}[u(\omega - X)] \leq u(\omega - \alpha)$$

---

no insurance \hspace{2cm} insurance

i.e.

$$p \cdot u(\omega - l) + [1 - p] \cdot u(\omega - 0) \leq u(\omega - \alpha)$$

---

no insurance \hspace{2cm} insurance

i.e.

$$\mathbb{E}[u(\omega - X)] \leq \mathbb{E}[u(\omega - \alpha - l + I)]$$

---

no insurance \hspace{2cm} insurance

Doherty & Schlessinger (1990) considered a model which integrates possible bankruptcy of the insurance company, but as an exogenous variable. Here, we want to make ruin endogenous.
Notations

\[ Y_i = \begin{cases} 
0 & \text{if agent } i \text{ claims a loss} \\
1 & \text{if not} 
\end{cases} \]

Let \( N = Y_1 + \cdots + Y_n \) denote the number of insured claiming a loss, and \( X = N/n \) denote the proportions of insured claiming a loss, \( F(x) = \mathbb{P}(X \leq x) \).

\[ \mathbb{P}(Y_i = 1) = p \text{ for all } i = 1, 2, \cdots, n \]

Assume that agents have identical wealth \( \omega \) and identical utility functions \( u(\cdot) \).

Further, insurance company has capital \( C = n \cdot c \), and ask for premium \( \alpha \).
Private insurance companies with limited liability

Consider $n = 5$ insurance policies, possible loss $\$1,000$ with probability 10%. Company has capital $C = 1,000$.

<table>
<thead>
<tr>
<th></th>
<th>Ins. 1</th>
<th>Ins. 1</th>
<th>Ins. 3</th>
<th>Ins. 4</th>
<th>Ins. 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Loss</td>
<td>-</td>
<td>1,000</td>
<td>-</td>
<td>1,000</td>
<td>-</td>
<td>2,000</td>
</tr>
</tbody>
</table>

Case 1: insurance company with limited liability

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<tbody>
<tr>
<td>indemnity</td>
<td>-</td>
<td>750</td>
<td>-</td>
<td>750</td>
<td>-</td>
<td>1,500</td>
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<tr>
<td>loss</td>
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<td>-250</td>
<td>-</td>
<td>-250</td>
<td>-</td>
<td>-500</td>
</tr>
<tr>
<td>net</td>
<td>-100</td>
<td>-350</td>
<td>-100</td>
<td>-350</td>
<td>-100</td>
<td>-1000</td>
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Possible government intervention

<table>
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<th>Ins. 3</th>
<th>Ins. 4</th>
<th>Ins. 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Loss</td>
<td>-</td>
<td>1,000</td>
<td>-</td>
<td>1,000</td>
<td>-</td>
<td>2,000</td>
</tr>
<tr>
<td>Case 2:</td>
<td></td>
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<tr>
<td>Tax</td>
<td>-100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>indemnity</td>
<td>-</td>
<td>1,000</td>
<td>-</td>
<td>1,000</td>
<td>-</td>
<td>2,000</td>
</tr>
<tr>
<td>net</td>
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<td>-200</td>
<td>-200</td>
<td>-200</td>
<td>-200</td>
<td>-1000</td>
</tr>
</tbody>
</table>

(note that it is a zero-sum game).
A one region model with homogeneous agents

Let $U(x) = u(\omega + x)$ and $U(0) = 0$.

Private insurance companies with limited liability:

- the company has a positive profit if $N \cdot \ell \leq n \cdot \alpha$

- the company has a negative profit if $n \cdot \alpha \leq N \cdot \ell \leq C + n \cdot \alpha$

- the company is bankrupted if $C + n \cdot \alpha \leq N \cdot \ell$

$\implies$ ruin of the insurance company if $X \geq \bar{x} = \frac{c + \alpha}{\ell}$

The indemnity function is

$$I(x) = \begin{cases} 
\ell & \text{if } x \leq \bar{x} \\
\frac{c + \alpha}{n} & \text{if } x > \bar{x}
\end{cases}$$
The diagram illustrates the probability of no ruin and the probability of ruin in the context of insurance covers of natural catastrophes on heterogeneous regions. The profit range is divided into two segments: positive profit and negative profit. The probability of no ruin is given by $F(\bar{x})$, where $\bar{x} = \frac{\alpha + c}{l}$, and the probability of ruin is $1 - F(\bar{x})$. The domain of the probability function is $[0; n\alpha]$. The critical value $\alpha$ and the deductible $c$ are denoted as $\alpha$ and $c$, respectively.
The objective function of the insured is $V$ defined as

$$E[E(U(-\alpha - \text{loss})|X)] = \int E(U(-\alpha - \text{loss})|X = x)dF(x)$$

where $E(U(-\alpha - \text{loss})|X = x)$ is equal to

$$P(\text{claim a loss}|X = x) \cdot U(\alpha - \text{loss}(x)) + P(\text{no loss}|X = x) \cdot U(-\alpha)$$

i.e.

$$E(U(-\alpha - \text{loss})|X = x) = x \cdot U(-\alpha - l + I(x)) + (1 - x) \cdot U(-\alpha)$$

so that

$$V = \int_0^1 [x \cdot U(-\alpha - l + I(x)) + (1 - x) \cdot U(-\alpha)]dF(x)$$

that can be written

$$V = U(-\alpha) - \int_0^1 x[U(-\alpha) - U(-\alpha - l + I(x))]f(x)dx$$

An agent will purchase insurance if and only if $V > p \cdot U(-l)$. 
with government intervention (or mutual fund insurance), the tax function is

\[
T(x) = \begin{cases} 
0 & \text{if } x \leq \bar{x} \\
\frac{N \ell - (\alpha + c)n}{n} = X \ell - \alpha - c & \text{if } x > \bar{x}
\end{cases}
\]

Then

\[
V = \int_0^1 \left[ x \cdot U(-\alpha - T(x)) + (1 - x) \cdot U(-\alpha - T(x)) \right] dF(x)
\]
i.e.

\[
V = \int_0^1 U(-\alpha + T(x))dF(x) = F(\bar{x}) \cdot U(-\alpha) + \int_{\bar{x}}^1 U(-\alpha - T(x))dF(x)
\]
A common shock model for natural catastrophes risks

Consider a possible natural catastrophe, modeled as an heterogeneous latent variable \( \Theta \), such that given \( \Theta \), the \( Y_i \)'s are independent, and

\[
\begin{align*}
\mathbb{P}(Y_i = 1|\Theta = \text{Catastrophe}) &= p_C \\
\mathbb{P}(Y_i = 1|\Theta = \text{No Catastrophe}) &= p_N
\end{align*}
\]

Let \( p^* = \mathbb{P}(\text{Cat}) \). Then the distribution of \( X \) is

\[
F(x) = \mathbb{P}(N \leq \lfloor nx \rfloor) = \mathbb{P}(N \leq k|\text{No Cat}) \times \mathbb{P}(\text{No Cat}) + \mathbb{P}(N \leq k|\text{Cat}) \times \mathbb{P}(\text{Cat})
\]

\[
= \sum_{j=0}^{k} \binom{n}{j} \left[ (p_N)^j (1 - p_N)^{n-j} (1 - p^*) + (p_C)^j (1 - p_C)^{n-j} p^* \right]
\]
Share of the population claiming a loss

Cumulative distribution function $F$

Probability density function $f$

$0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0$

$0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0$

$0.0 \quad 5 \quad 10 \quad 15 \quad 20$

$\frac{1-p^*}{p}$

$p_N \quad p \quad p_C$
Probability density function $f$

Cumulative distribution function $F$

Share of the population claiming a loss

$1 - p^*$

$p_C$

$p_N$

$p$
Equilibriums in the Expected Utility framework

The expected profit of the insurance company is

$$\Pi(\alpha, c) = \int_0^\bar{x} [n\alpha - xn\ell] dF(x) - [1 - F(\bar{x})]cn$$

(1)

A premium smaller than the pure premium can lead to a positive expected profit. In Rothschild & Stiglitz (QJE, 1976) a positive profit was obtained if and only if $\alpha > p \cdot l$. Here companies have limited liabilities.

If agents are risk adverse, for a given premium $\alpha$, their expected utility is always higher with government intervention.

Proof. Risk adverse agents look for mean preserving spread lotteries.  \qed
Expected utility

\[ pU(-l) = -63.9 \]

Expected profit<0

Expected profit>0

Premium

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The two region model

Consider here a two-region chock model such that

- $\Theta = (0, 0)$, no catastrophe in the two regions,
- $\Theta = (1, 0)$, catastrophe in region 1 but not in region 2,
- $\Theta = (0, 1)$, catastrophe in region 2 but not in region 1,
- $\Theta = (1, 1)$, catastrophe in the two regions.

Let $N_1$ and $N_2$ denote the number of claims in the two regions, respectively, and set $N_0 = N_1 + N_2$. 
The two region model

\[
X_1 \sim F_1(x_1|p_1, \delta_1) = F_1(x_1), \\
X_2 \sim F_2(x_2|p_2, \delta_2) = F_2(x_2), \\
X_0 \sim F_0(x_0|F_1, F_2, \theta) = F_0(x_0|p_1, p_2, \delta_1, \delta_2, \theta) = F_0(x_0),
\]

Note that there are two kinds of correlation in this model,

- a **within** region correlation, with coefficients \(\delta_1\) and \(\delta_2\)
- a **between** region correlation, with coefficient \(\delta_0\)

Here, \(\delta_i = 1 - p_i^N/p_i^C\), where \(i = 1, 2\) (Regions), while \(\delta_0 \in [0, 1]\) is such that

\[
P(\Theta = (1, 1)) = \delta_0 \times \min\{P(\Theta = (1, \cdot)), P(\Theta = (\cdot, 1))\} = \delta_0 \times \min\{p_1^*, p_2^*\}.
\]
The two region model

The following graphs show the decision in Region 1, given that Region 2 buy insurance (on the left) or not (on the right).
The two region model

The following graphs show the decision in Region 2, given that Region 1 buy insurance (on the left) or not (on the right).
In a **Strong Nash equilibrium** which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy unilaterally.
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Possible Nash equilibriums
Possible Nash equilibriums