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<http://freakonometrics.hypotheses.org/>

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Welfare, Inequality & Poverty, # 4

Regression?

Galton (1870, galton.org, 1886, galton.org) and Pearson & Lee (1896, jstor.org, 1903 jstor.org) studied genetic transmission of characteristics, e.g. the height.

On average the child of tall parents is taller than other children, but less than his parents.

“I have called this peculiarity by the name of regression’, Francis Galton, 1886.

REGRESSION *towards* MEDIOCRITY in HEREDITARY STATURE. By FRANCIS GALTON, F.R.S., &c.

Table 8.1. Galton's 1885 cross-tabulation of 928 adult children born of 205 midparents, by their height and their midparent's height.

Height of the mid-parent in inches	Height of the adult child											Total no. of adult children	Total no. of mid-parents	Medians			
	<61.7	62.2	63.2	64.2	65.2	66.2	67.2	68.2	69.2	70.2	71.2				72.2	73.2	>73.7
> 73.0	—	—	—	—	—	—	—	—	—	—	—	1	3	—	4	5	—
72.5	—	—	—	—	—	—	—	—	—	1	2	1	2	2	4	19	6
71.5	—	—	—	—	1	3	4	5	5	10	4	9	2	2	43	111	69.8
70.5	1	—	1	—	1	1	3	12	18	14	7	4	3	3	68	22	69.5
69.5	—	—	1	16	4	17	27	20	33	25	20	11	4	5	183	41	68.9
68.5	1	—	7	11	16	25	31	34	48	21	18	4	3	—	219	49	68.2
67.5	—	3	5	14	15	36	58	28	38	19	11	4	—	—	211	53	67.6
66.5	—	3	5	2	17	17	14	13	4	—	—	—	—	—	78	20	67.2
65.5	1	—	9	5	7	11	11	7	7	5	2	1	—	—	66	12	66.7
64.5	1	1	4	4	1	5	5	—	2	—	—	—	—	—	25	5	65.8
<64.0	1	—	2	4	1	2	2	1	1	—	—	—	—	—	14	1	—
Totals	5	7	32	59	48	117	158	120	167	99	64	41	17	14	928	205	—
Medians	—	—	66.3	67.8	67.9	67.7	67.9	68.3	68.5	69.0	69.0	70.0	—	—	—	—	—

Source: Galton (1886).
 Note: All female heights were multiplied by 1.08 before tabulation. Galton added an explanatory footnote to the table: "In calculating the Medians, the entries have been taken as referring to the middle of the squares in which they stand. The reason why the headings run 62.2, 63.2, &c., instead of 62.5, 63.5, &c., is that the observations are unequally distributed between 62 and 63, 63 and 64, &c., there being a strong bias in favour of integral inches. After careful consideration, I concluded that the headings, as adopted, best satisfied the conditions. This inequality was not apparent in the case of the Mid-parents." Galton republished these data in 1889, where they are referred to as the R.F.F. Data (Record of Family Faculties); he then noted that the first row must be in error (four children cannot have five sets of parents), but he claimed that "the bottom line, which looks suspicious is correct" (p. 208).

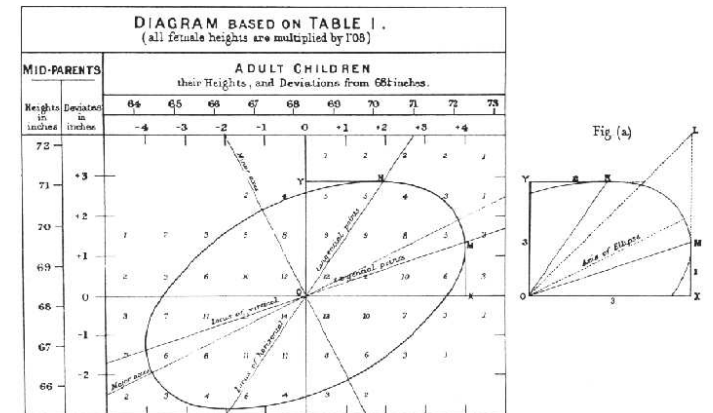
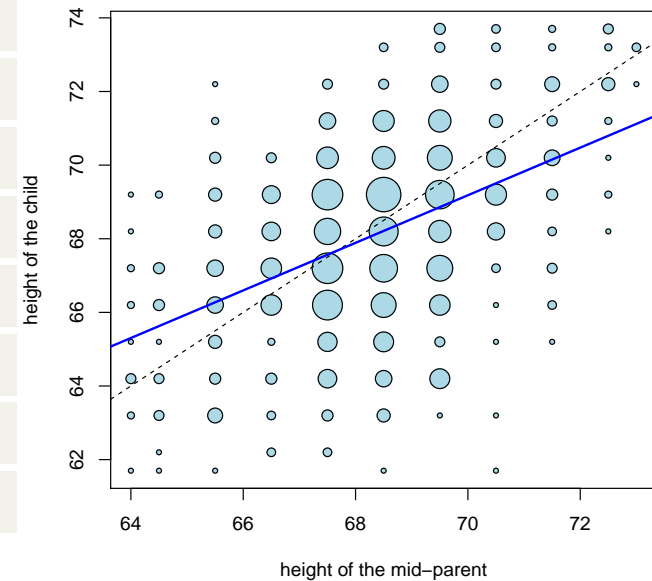


Figure 8.7. Galton's smoothed rendition of Table 8.1, with one of the "concentric and similar ellipses" drawn in. The geometric relationship of the two regression lines to the ellipse is also shown. (From Galton, 1886a.)

Regression?

```
1 > library(HistData)
2 > attach(Galton)
3 > Galton$count <- 1
4 > df <- aggregate(Galton, by=list(parent,
  child), FUN=sum)[,c(1,2,5)]
5 > plot(df[,1:2], cex=sqrt(df[,3]/3))
6 > abline(a=0,b=1,lty=2)
7 > abline(lm(child~parent, data=Galton))
```



Least Squares?

Recall that

$$\left\{ \begin{array}{l} \mathbb{E}(Y) = \operatorname{argmin}_{m \in \mathbb{R}} \left\{ \|Y - m\|_{\ell_2}^2 = \mathbb{E}([Y - m]^2) \right\} \\ \operatorname{Var}(Y) = \min_{m \in \mathbb{R}} \left\{ \mathbb{E}([Y - m]^2) \right\} = \mathbb{E}([Y - \mathbb{E}(Y)]^2) \end{array} \right.$$

The empirical version is

$$\left\{ \begin{array}{l} \bar{y} = \operatorname{argmin}_{m \in \mathbb{R}} \left\{ \sum_{i=1}^n \frac{1}{n} [y_i - m]^2 \right\} \\ s^2 = \min_{m \in \mathbb{R}} \left\{ \sum_{i=1}^n \frac{1}{n} [y_i - m]^2 \right\} = \sum_{i=1}^n \frac{1}{n} [y_i - \bar{y}]^2 \end{array} \right.$$

The conditional version is

$$\left\{ \begin{array}{l} \mathbb{E}(Y|\mathbf{X}) = \operatorname{argmin}_{\varphi: \mathbb{R}^k \rightarrow \mathbb{R}} \left\{ \|Y - \varphi(\mathbf{X})\|_{\ell_2}^2 = \mathbb{E}([Y - \varphi(\mathbf{X})]^2) \right\} \\ \operatorname{Var}(Y|\mathbf{X}) = \min_{\varphi: \mathbb{R}^k \rightarrow \mathbb{R}} \left\{ \mathbb{E}([Y - \varphi(\mathbf{X})]^2) \right\} = \mathbb{E}([Y - \mathbb{E}(Y|\mathbf{X})]^2) \end{array} \right.$$

Changing the Distance in Least-Squares?

One might consider $\hat{\beta} \in \operatorname{argmin} \left\{ \sum_{i=1}^n |Y_i - \mathbf{X}_i^\top \beta| \right\}$, based on the ℓ_1 -norm, and not the ℓ_2 -norm.

This is the **least-absolute deviation** estimator, related to the **median regression**, since $\operatorname{median}(X) = \operatorname{argmin} \{ \mathbb{E}|X - x| \}$.

More generally, assume that, for some function $R(\cdot)$,

$$\hat{\beta} \in \operatorname{argmin} \left\{ \sum_{i=1}^n R(Y_i - \mathbf{X}_i^\top \beta) \right\}$$

If R is differentiable, the first order condition would be

$$\sum_{i=1}^n R' \left(Y_i - \mathbf{X}_i^\top \beta \right) \cdot \mathbf{X}_i^\top = 0.$$

Changing the Distance in Least-Squares?

i.e.

$$\sum_{i=1}^n \underbrace{\omega \left(Y_i - \mathbf{X}_i^\top \boldsymbol{\beta} \right)}_{\omega_i} \cdot \left(Y_i - \mathbf{X}_i^\top \boldsymbol{\beta} \right) \mathbf{X}_i^\top = 0 \text{ with } \omega(x) = \frac{R'(x)}{x},$$

It is the first order condition of a **weighted ℓ_2 regression**.

To obtain the ℓ_1 -regression, observe that $\omega = |\varepsilon|^{-1}$

Changing the Distance in Least-Squares?

⇒ use iterative (weighted) least-square regressions.

Start with some standard ℓ_2 regression

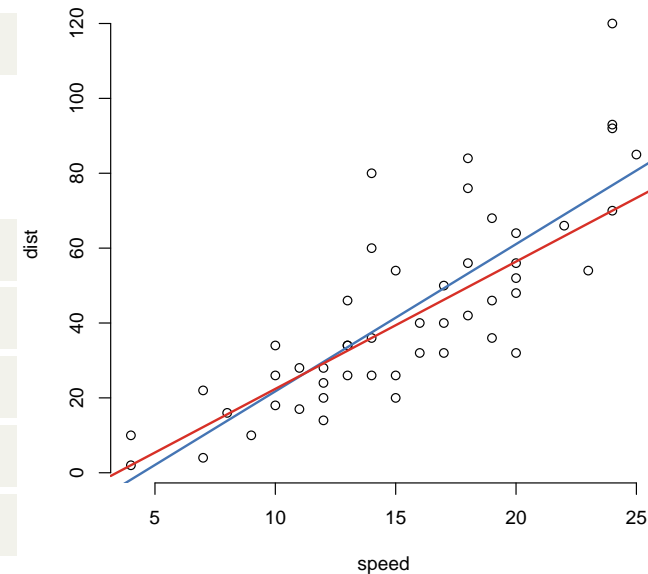
```
1 > reg_0 <- lm(Y~X, data=db)
```

For the ℓ_1 regression consider weight function

```
1 > omega <- function(e) 1/abs(e)
```

Then consider the following iterative algorithm

```
1 > resid <- residuals(reg_0)
2 > for(i in 1:100){
3 + W <- omega(e)
4 + reg <- lm(Y~X, data=db, weights=W)
5 + e <- residuals(reg)}
```



Quantile Regression

Observe that, for all $\tau \in (0, 1)$

$$Q_X(\tau) = F_X^{-1}(\tau) = \operatorname{argmin}_{m \in \mathbb{R}} \{\mathbb{E}[R_\tau(X - m)]\}$$

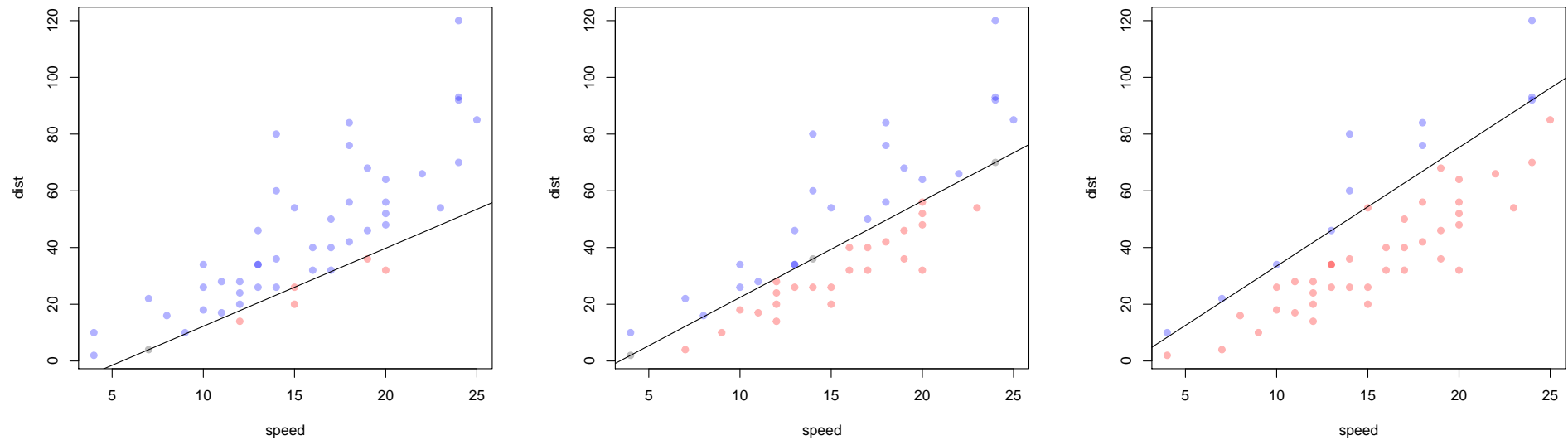
where $R_\tau(x) = [\tau - \mathbf{1}(x < 0)] \cdot x$.

From a statistical point of view

$$\hat{Q}_x(\tau) = \operatorname{argmin}_{m \in \mathbb{R}} \left\{ \frac{1}{n} \sum_{i=1}^n R_\tau(x_i - m) \right\}.$$

The quantile- τ regression

$$\hat{\beta} = \operatorname{argmin} \left\{ \sum_{i=1}^n \mathcal{R}_\tau(Y_i - \mathbf{X}_i^\top \beta) \right\}.$$



There are $n(1 - p)$ points in the upper region, and np in the lower one.

```
1 > library(quantreg)
2 > fit1 <- rq(y ~ x1 + x2, tau = .1, data = df)
```

see cran.r-project.org.

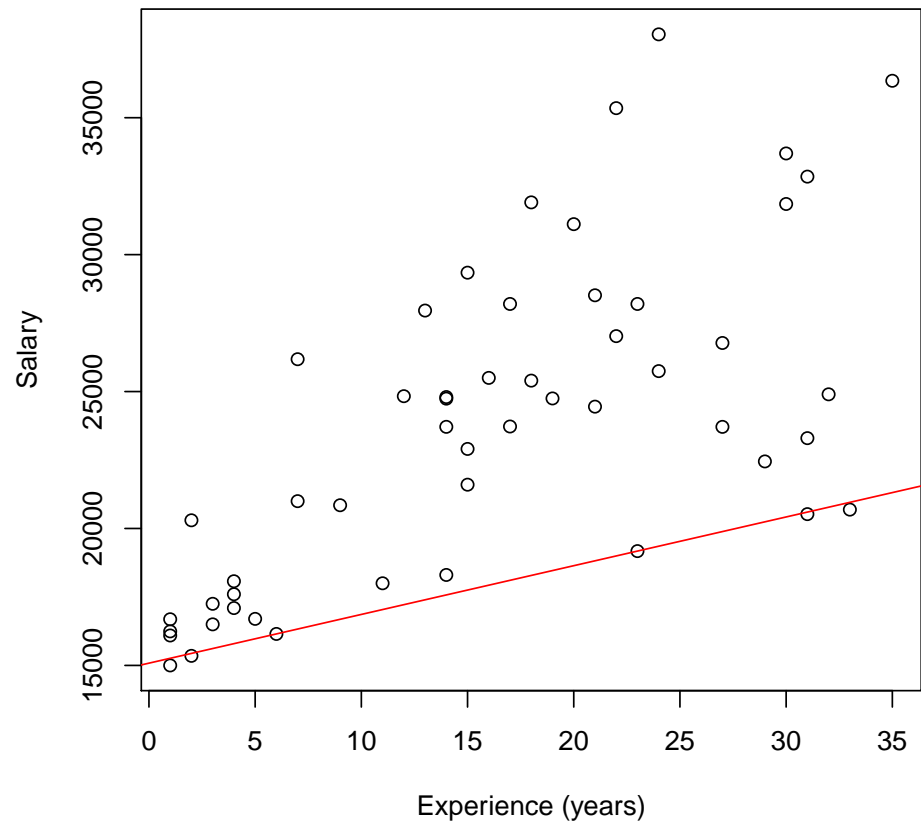
Quantile Regression: Empirical Analysis

Consider here some salaries, as a function of the experience (in years), see data.princeton.edu

```

1 > salary=read.table("http://data.princeton.edu/wws509/datasets/salary.dat",header=TRUE)
2 > library(quantreg)
3 > plot(salary$yd,c)
4 > abline(rq(sl~yd,tau=.1,data=salary),col="red")

```

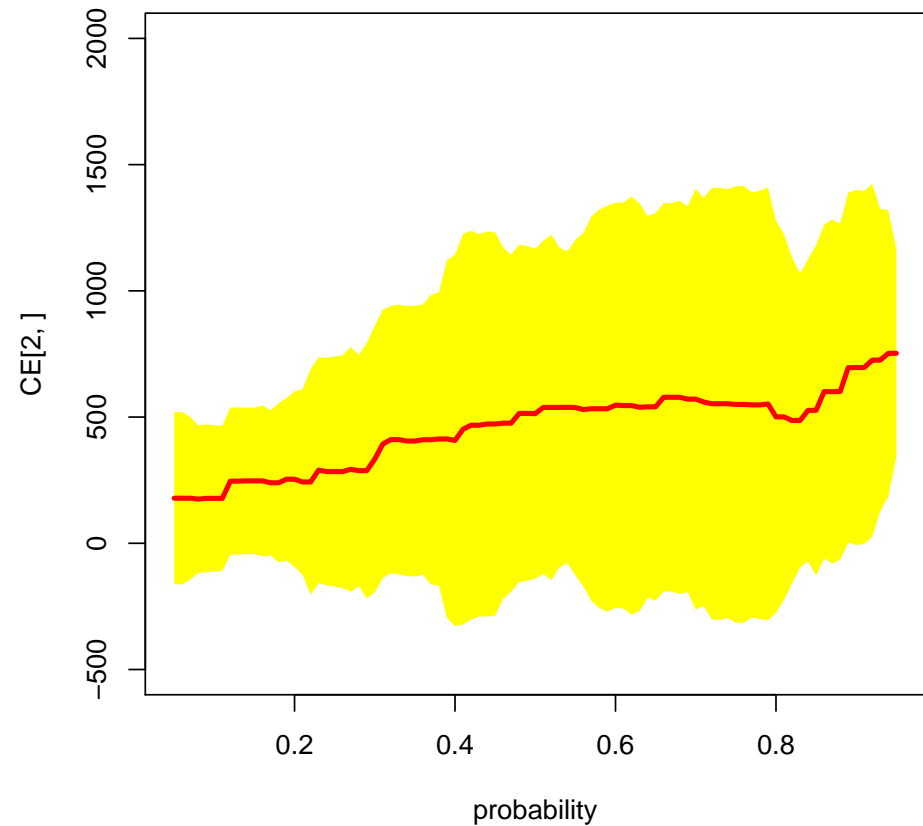


Quantile Regression: Empirical Analysis

```

1 > u <- seq(.05, .95, by=.01)
2 > coefstd <- function(u) summary(
  rq(sl~yd, data=salary, tau=u))$
  coefficients[,2]
3 > coefest <- function(u) summary(
  rq(sl~yd, data=salary, tau=u))$
  coefficients[,1]
4 > CS <- Vectorize(coefstd)(u)
5 > CE <- Vectorize(coefest)(u)
6 > CEinf <- CE-2*CS
7 > CEsup <- CE+2*CS
8 > plot(u, CE[2,], ylim=c(-500,2000)
  , col="red")
9 > polygon(c(u, rev(u)), c(CEinf
  [2,], rev(CEsup[2,])), col="
  yellow", border=NA)

```



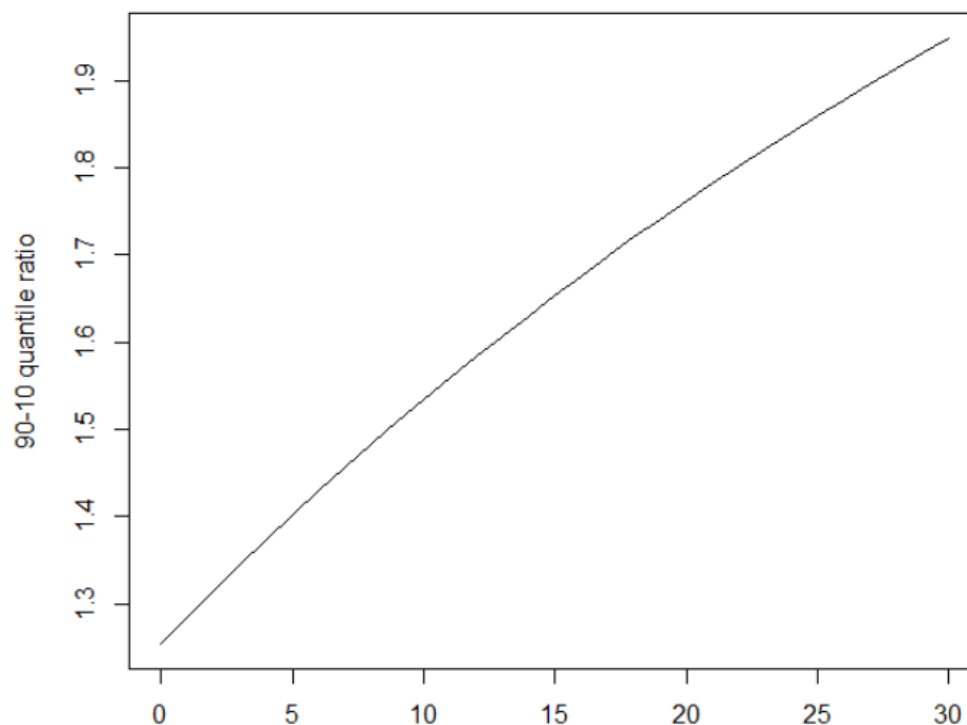
Quantile Regression: Empirical Analysis

Consider the evolution of the 90%–10% quantile ratio,

```

1 > ratio9010 = function(age) {
2 +   predict(Q90, newdata=data.
3 +   frame(yd=age)) /
4 +   predict(Q10, newdata=data.
5 +   frame(yd=age))
6 + }
7 > ratio9010(5)
8 1.401749
9 > A=0:30
10 > plot(A, Vectorize(ratio9010)(A),
11 + type="l", ylab="90-10 quantile
12 + ratio")

```



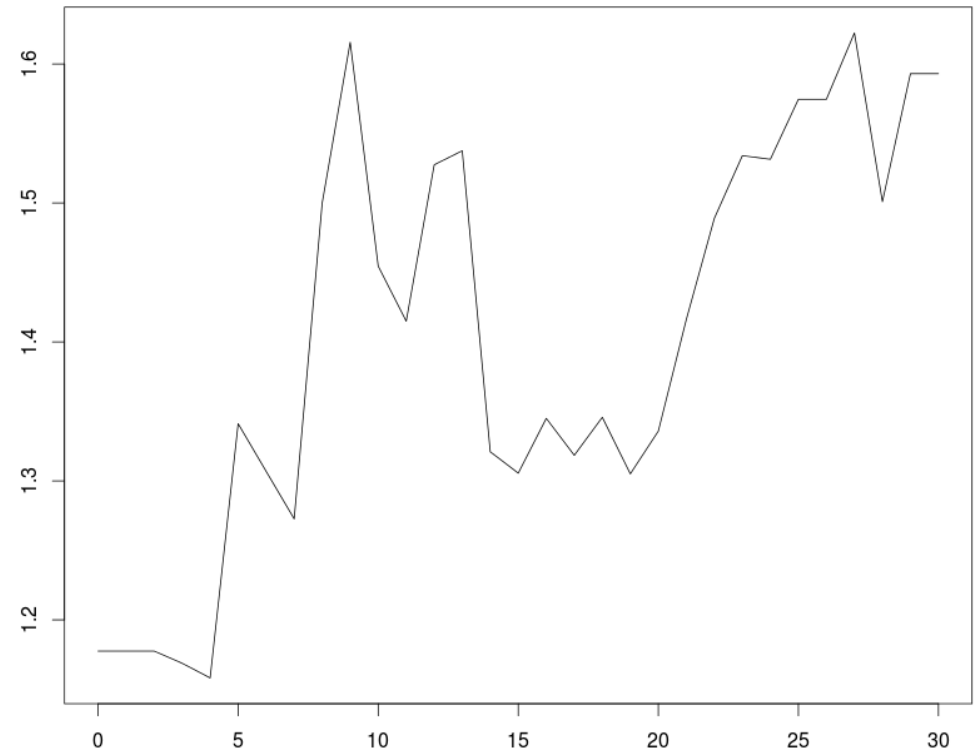
Local Regression: Empirical Analysis

which is smoother than the local estimator

```

1 > ratio9010_k = function (age, k
  =10){
2 +   idx=which (rank (abs (salary$yd-
  age)) <=k)
3 +   quantile (salary$sl [idx] ,.9) /
  quantile (salary$sl [idx] ,.1) }
4 > A=0:30
5 > plot (A, Vectorize (ratio9010_k) (A
  ), type="l" , ylab="90-10
  quantile ratio ")

```

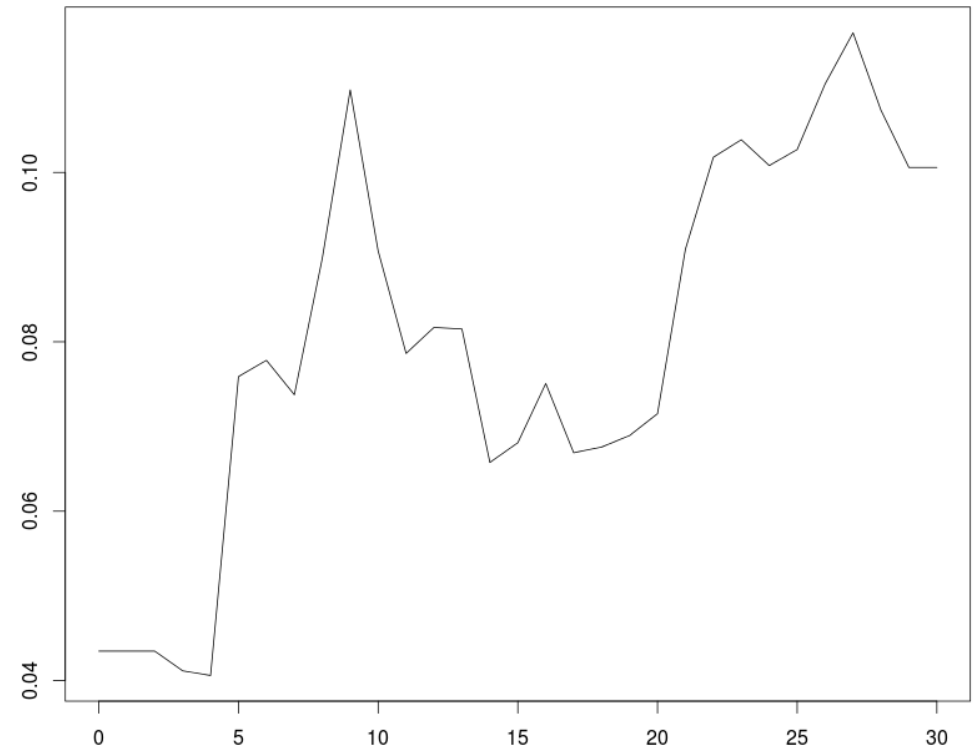


Local Regression: Empirical Analysis

```
1 > Gini(salary$s1)
2 [1] 0.1391865
```

We can also consider some local Gini index

```
1 > Gini_k = function(age, k=10){
2 +   idx=which(rank(abs(salary$yd-
3 +   Gini(salary$s1[idx]))}
4 > A=0:30
5 > plot(A, Vectorize(Gini_k)(A),
        type="l", ylab="Local Gini
        index")
```



Datasets for Empirical Analysis

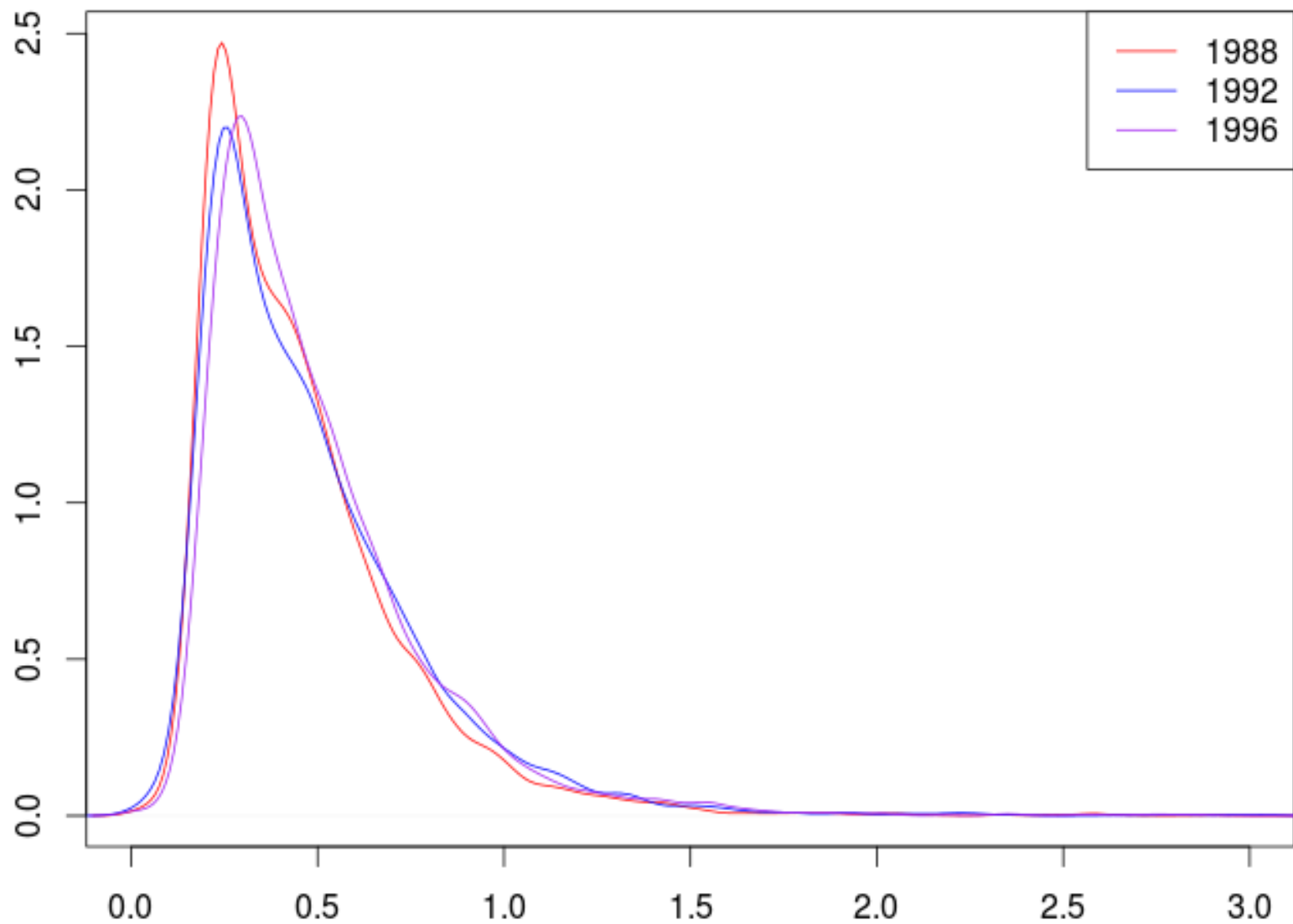
Income the U.K., in 1988, 1992 and 1996,

```
1 > uk88 <- read.csv("http://www.vcharite.univ-mrs.fr/pp/lubrano/cours/
  fes88.csv", sep=";", header=FALSE) $V1
2 > uk92 <- read.csv("http://www.vcharite.univ-mrs.fr/pp/lubrano/cours/
  fes92.csv", sep=";", header=FALSE) $V1
3 > uk96 <- read.csv("http://www.vcharite.univ-mrs.fr/pp/lubrano/cours/
  fes96.csv", sep=";", header=FALSE) $V1
4 > cpi <- c(421.7, 546.4, 602.4)

5 > y88 <- uk88 / cpi [1]
6 > y92 <- uk92 / cpi [2]
7 > y96 <- uk96 / cpi [3]

8 > plot(density(y88), type="l", col="red")
9 > lines(density(y92), type="l", col="blue")
10 > lines(density(y96), type="l", col="purple")
```

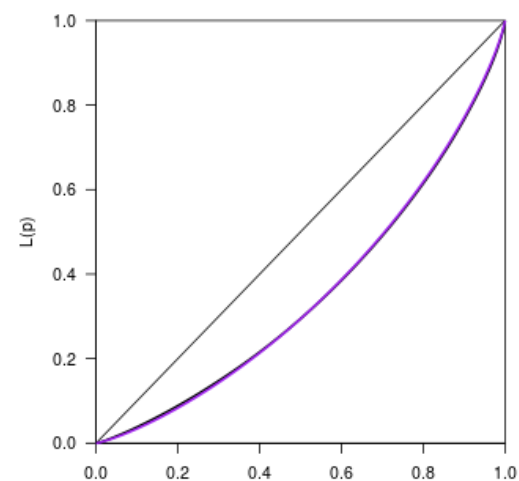
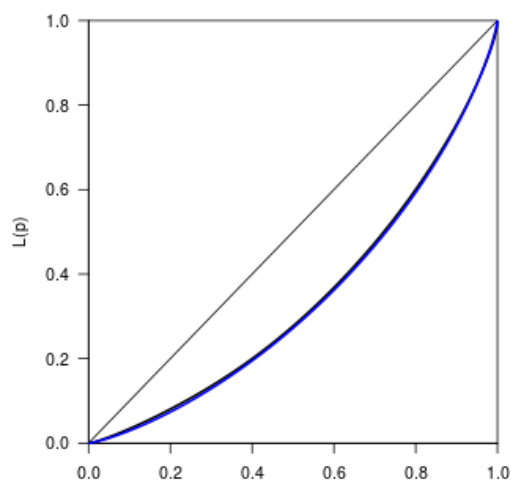
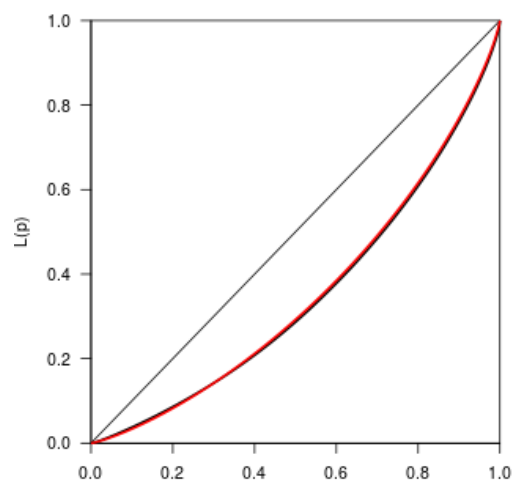
Datasets for Empirical Analysis



Inequalities: Empirical Analysis

We can visualize empirical Lorenz curves, and theoretical version (lognormal)

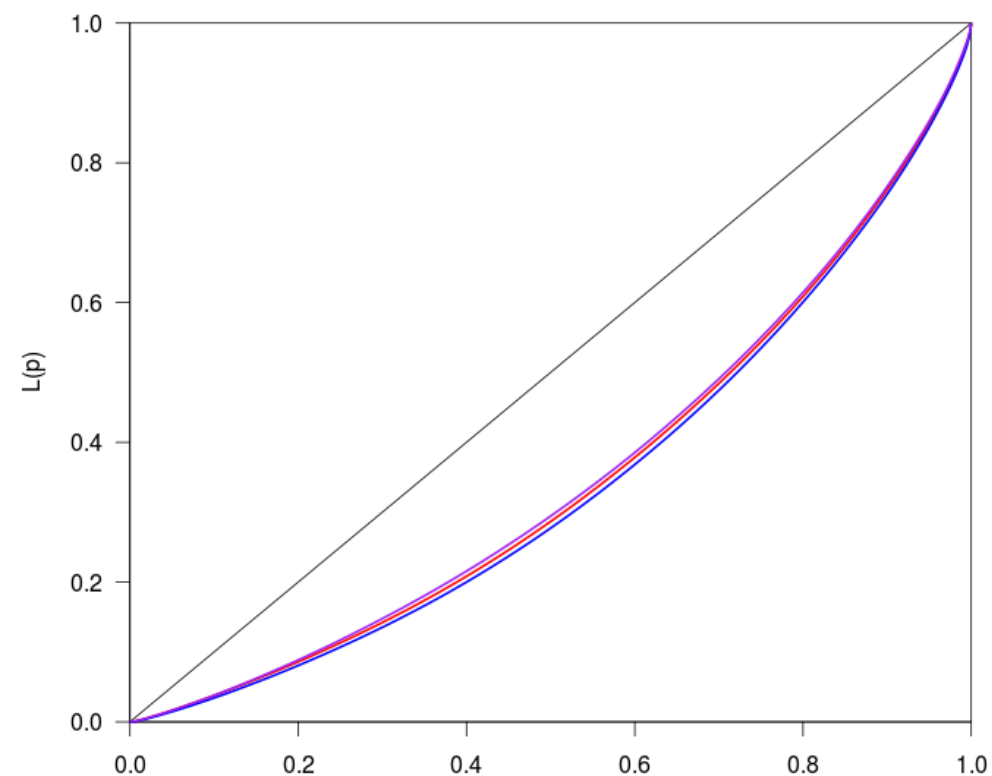
```
1 > plot(Lc(y88)); s=sd(log(y88)); lines(Lc.lognorm, parameter=s)
2 > plot(Lc(y92)); s=sd(log(y92)); lines(Lc.lognorm, parameter=s)
3 > plot(Lc(y96)); s=sd(log(y96)); lines(Lc.lognorm, parameter=s)
```



Inequalities: Empirical Analysis

If we plot the three curves on the same graph,

```
1 > plot(Lc(y88), col="red")  
2 > lines(Lc(y92), col="blue")  
3 > lines(Lc(y96), col="purple")
```

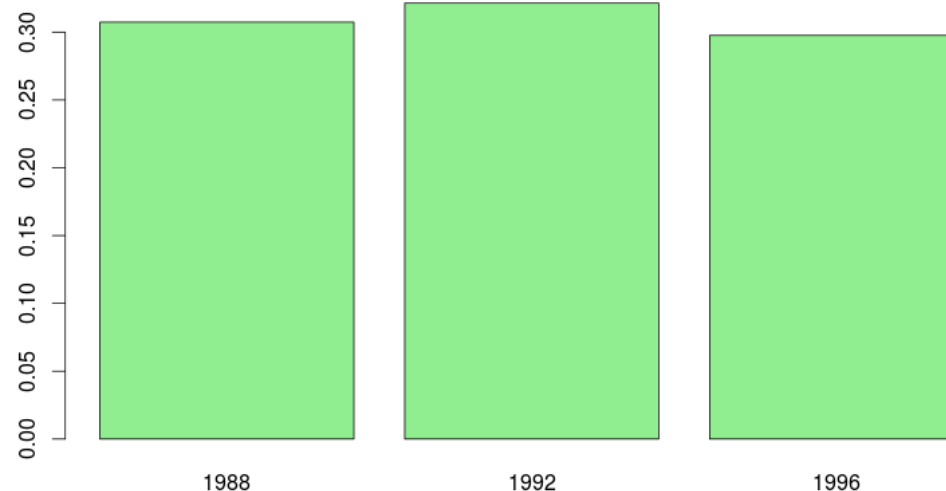


Inequalities: Empirical Analysis

```

1 > inequalities=function(f_ineq){
2 +   z88 =f_ineq(y88); z92 = f_ineq
   (y92); z96 = f_ineq(y96)
3 +   I=cbind(z88 ,z92 ,z96)
4 +   names(I)=c("1988" ,"1992" ,"1996"
   ")
5 +   cat(" 1988..." ,z88 ,"\n 1992..."
   " ,z92 ,"\n 1996..." ,z96 ,"\n")
6 +   barplot(I ,col="light green" ,
   names.arg=c("1988" ,"1992" ,"
   1996" ))
7 +   return(I)}
8 > I<-inequalities(Gini)
9   1988... 0.3073511
10  1992... 0.3214023
11  1996... 0.2975789

```



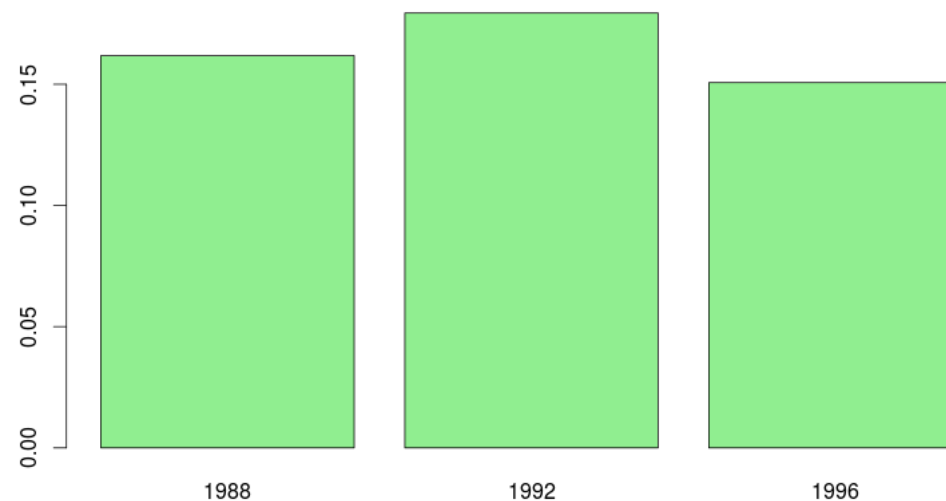
Inequalities: Empirical Analysis

```
1 > I<-inequalities(Theil)
```

```
2 1988... 0.1618547
```

```
3 1992... 0.1794008
```

```
4 1996... 0.1506973
```



Welfare Functions

A **welfare function** as a function with n arguments $W(\mathbf{x}) = W(x_1, \dots, x_n)$.

Assume that W is normalized, so that $W(\mathbf{1}) = 1$.

It represents social preferences over the income distribution, and it should satisfy some axioms,

Pareto axiom: The welfare function is increasing for all its inputs

$$W(\mathbf{x} + \boldsymbol{\epsilon}) \geq W(\mathbf{x}) \text{ for all } \boldsymbol{\epsilon} \geq 0.$$

Symmetry axiom or anonymity: We can permute the individuals without changing the value of the function

$$W(x_1, x_2, \dots, x_n) = W(x_2, x_1, \dots, x_n)$$

Welfare Functions

Principle of transfers: the quasi concavity of the welfare function implies that if we operate a monetary transfer from a rich to a poor, welfare is increased, provided that the transfer does not modify the ordering of individuals (Pigou-Dalton principle)

$$W(x_1, \dots, x_i, \dots, x_j, \dots, x_n) \geq W(x_1, \dots, x_i + \delta, \dots, x_j - \delta, \dots, x_n)$$

Other axioms can be added, e.g. **homogeneous of order 1**,

$$W(\lambda \mathbf{x}) = \lambda W(\mathbf{x}) \text{ for all } \lambda \geq 0.$$

Thus (all homogeneous function of order 1 can be defined on the simplex)

$$W(\mathbf{x}) = \bar{x} \cdot W\left(\frac{\mathbf{x}}{\bar{x}}\right) \text{ for all } \lambda \geq 0.$$

Welfare Functions

Observe that $W(\bar{x}\mathbf{1}) = \bar{x}$. And because of the aversion for inequality, $W(\mathbf{x}) \leq \bar{x}$.
One can denote

$$W(\mathbf{x}) = \bar{x} \cdot [1 - I(\mathbf{x})]$$

for some function $I(\cdot)$, which takes values in $[0, 1]$.

$I(\cdot)$ is then interpreted as an inequality measure and $\bar{x} \cdot I(\mathbf{x})$ represents the (social) cost of inequality.

See fao.org.

Welfare Functions

E.g. **utilitarian** (or **Benthamian**) function

$$W(\mathbf{x}) = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

“*dollar is a dollar*” approach: no inequality aversion.

E.g. **Rawlsian** welfare function

$$W(\mathbf{x}) = \min\{y_1, \dots, y_n\}.$$

Social welfare cannot increase unless the income of the poorest individual is increased: infinite inequality aversion.

From Welfare Functions to Inequality Indices

Consider the standard welfare function,

$$W(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^n \frac{x_i^{1-\epsilon}}{1-\epsilon}$$

with the limiting case (where $\epsilon \rightarrow 1$)

$$W(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^n \log(x_i)$$

When $\epsilon \rightarrow 1$ we have the **Benthamian** function, and when $\epsilon \rightarrow \infty$, we have the **Rawlsian** function. Thus, ϵ can be interpreted as an inequality aversion parameter.

The ratio of marginal social utilities of two individuals i and j has a simple expression

$$\frac{\partial W / \partial x_i}{\partial W / \partial x_j} = \left(\frac{x_i}{x_j} \right)^{-\epsilon}$$

When ϵ increases, the marginal utility of the poorest dominates, see Rawls (1971) [wikipedia.org](https://en.wikipedia.org/wiki/John_Rawls), the objective of the society is to maximise the situation of the poorest.

From that welfare function, define the implied inequality index,

$$I = 1 - \left(\frac{1}{n} \sum_{i=1}^n \left(\frac{x_i}{\bar{x}} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

which is Atkinson index.

Equally Distributed Equivalent

Given \mathbf{x} define ξ (or $\xi(\mathbf{x})$) as

$$W(\xi \mathbf{1}) = W(\mathbf{x})$$

From the principle of transfers, $\xi \leq \bar{x}$. Then one can define

$$I(\mathbf{x}) = 1 - \frac{\xi(\mathbf{x})}{\bar{x}}.$$

If $I(\cdot)$ satisfies the scale independence axiom, $I(\mathbf{x}) = I(\lambda \mathbf{x})$, then

$$\xi(\mathbf{x}) = \left(\frac{1}{n} \sum_{i=1}^n (x_i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

This index has a simple interpretation : if $I = 0.370\%$ of the total income is necessary to reach the same value of welfare, provided that income is equally distributed.

Kolm (1976) suggested that the welfare function should not change if the same positive amount is given to everybody, i.e.

$$W(\mathbf{x}) = W(\mathbf{x} + h\mathbf{1})$$

This leads to

$$I(\mathbf{x}) = \frac{1}{\alpha} \log \left(\frac{1}{n} \sum_{i=1}^n \exp[\alpha(x_i - \bar{x})] \right)$$

From Inequality Indices to Welfare Functions

Consider e.g. Gini index

$$G(\mathbf{x}) = \frac{2}{n(n-1)\bar{x}} \sum_{i=1}^n i \cdot x_{i:n} - \frac{n+1}{n-1}$$

$$G(\mathbf{x}) = \frac{1}{2n^2\bar{x}} \sum_{i,j=1}^n |x_i - x_j|$$

then define

$$W(\mathbf{x}) = \bar{x} \cdot [1 - G(\mathbf{x})]$$

as suggested in Sen (1976, [jstor.org](https://www.jstor.org))

More generally, consider

$$W(\mathbf{x}) = \bar{x} \cdot [1 - G(\mathbf{x})]^\sigma$$

with $\sigma \in [0, 1]$.

From Inequality to Poverty

an absolute line of poverty is defined with respect to a minimum level of subsistence

In developed countries and more precisely within the EU, one prefer to define a relative poverty line, defined with respect to a fraction of the mean or the median of the income distribution.

The **headcount ratio** evaluates the number of poor (below a threshold z)

$$H(\mathbf{x}, z) = \frac{1}{n} \sum_{i=1}^n \mathbf{1}(x_i \leq z) = \hat{F}(z) = \frac{q}{n}$$

where q is the number of poors.

The **income gap ratio** $I(\mathbf{x}, z)$ measures in percentage the gap between the poverty line z and the mean income among the poor

$$I(\mathbf{x}, z) = \frac{1}{z} \left(z - \frac{1}{q} \sum_{i=1}^n x_i \mathbf{1}(x_i \leq z) \right) = \frac{1}{z} \left(z - \frac{1}{q} \sum_{i=1}^q x_{i:n} \right) = 1 - \frac{\mu_p}{z}$$

where μ_p is the average income of the poor.

The **poverty gap ratio** is defined as

$$HI(\mathbf{x}, z) = \frac{q}{n} \left(1 - \frac{1}{qz} \sum_{i=1}^q x_{i:n} \right)$$

Watts (1968) suggested also

$$W(\mathbf{x}, z) = \frac{1}{q} \sum_{i=1}^q [\log z - \log x_{i:n}]$$

which can be written

$$W = H \cdot (T - \log(1 - I))$$

where T is Theil index (Generalize Entropy, with index 1)

$$T = \frac{1}{n} \sum_{i=1}^n \frac{x_i}{\bar{x}} \log \left(\frac{x_i}{\bar{x}} \right).$$

```
1 > Watts(x, z, na.rm = TRUE)
```

Sen Poverty Indices

$$S(\mathbf{x}, z) = H(\mathbf{x}, z) \cdot [I(\mathbf{x}, z) + [1 - (\mathbf{x}, z)]G_p]$$

where G_p is Gini index of the poors.

- if $G_p = 0$ then $S = HI$
- if $G_p = 1$ then $S = H$

```
1 > Sen(x, z, na.rm = TRUE)
```

One can write

$$S = \frac{2}{(q+1)nz} \sum_{i=1}^q [z - x_{i:n}][q+1-i]$$

Thon (1979) suggested a similar expression, but with (slightly) different weights

$$\text{Thon} = \frac{2}{n(n+1)z} \sum_{i=1}^q [z - x_{i:n}][n+1-i]$$

But it suffers some drawbacks: it violates the principle of transfers and is not continuous in \mathbf{x} . Shorrocks (1995, [jstor.org](https://www.jstor.org)) suggested

$$SST(\mathbf{x}, z) = [2 - H(\mathbf{x}, z)] \cdot H(\mathbf{x}, z) \cdot I(\mathbf{x}, z) + H(\mathbf{x}, z)^2 [1 - I(\mathbf{x}, z)] \cdot G_P$$

Observe that Sen index is defined as

$$S = \frac{2}{(q+1)n} \sum_{i=1}^q \underbrace{\frac{z - x_{i:n}}{z}}_{\tilde{x}_i} [q + 1 - i]$$

while

$$SST = \frac{1}{n^2} \sum_{i=1}^q \underbrace{\frac{z - x_{i:n}}{z}}_{\tilde{x}_i} [2n - 2i + 1]$$

This index is symmetric, monotonic, homogeneous of order 0 and takes values in $[0, 1]$. Further it is continuous and consistent with the transfert axiom.

On can write

$$SST = \bar{\tilde{x}} \cdot [1 - G(\tilde{\mathbf{x}})].$$

```
1 > SST(x, z, na.rm = TRUE)

1 > poverty=function(f_pov, z_fun=function(x) mean(x) / 2, ...) {
2 +   z88 =z_fun(y88); z92 = z_fun(y92); z96 = z_fun(y96)
3 +   p88=f_pov(y88, z88); p92=f_pov(y92, z92); p96=f_pov(y96, z96)
4 +   P=cbind(p88, p92, p96)
5 +   names(P)=c("1988", "1992", "1996")
6 +   cat(" 1988...", p88, "\n 1992...", p92, "\n 1996...", p96, "\n")
7 +   barplot(P, col="light green", names.arg=c("1988", "1992", "1996"))
8 +   return(P)}
```

FGT Poverty Indices

Foster, Greer & Thorbecke (1984, darp.lse.ac.uk) suggested a class of poverty indices that were decomposable,

$$P_{\alpha}(\mathbf{x}, z) = \frac{1}{n} \sum_{i=1}^q \left(1 - \frac{x_i}{z}\right)^{\alpha}$$

where $\alpha \in \{0, 1, 2, \dots\}$.

When $\alpha = 0$ we get the headcount measure,

$$P_0 = \frac{1}{n} \sum_{i=1}^q \mathbf{1}(x_i \leq z) = \frac{q}{n}$$

When $\alpha = 1$ we get an average of poverty gap $z - x_i$

$$P_1 = \frac{1}{n} \sum_{i=1}^q \left(1 - \frac{x_i}{z}\right) \mathbf{1}(x_i \leq z)$$

(see HI).

In R, the parameter is $1 + \alpha$

```
1 > Foster(x, k, parameter = 1, na.rm = TRUE)
```

i.e. it gives for parameter 1 the headcount ratio and for parameter 2 the poverty gap ratio.

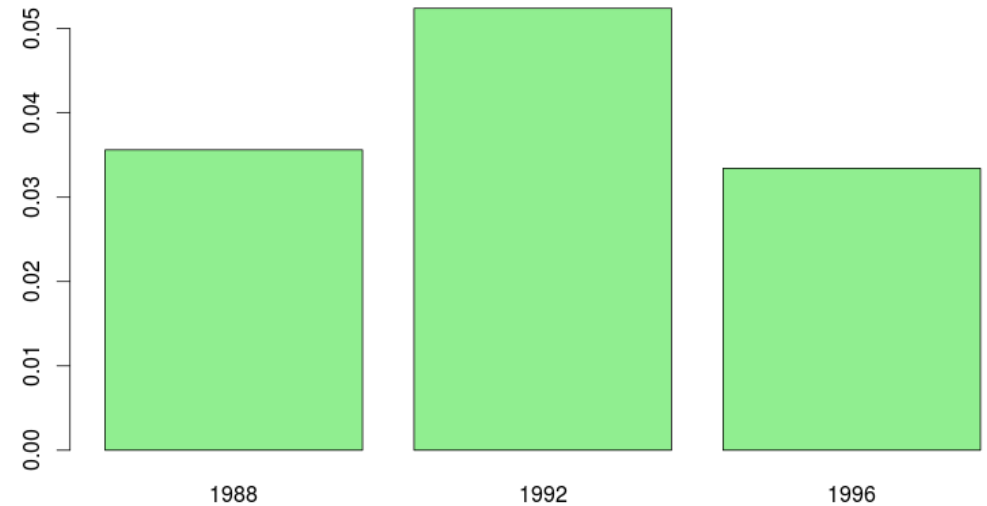
When $\alpha = 2$

$$P_2 = \frac{1}{n} \sum_{i=1}^q \left(1 - \frac{x_i}{z}\right)^2 \mathbf{1}(x_i \leq z)$$

Inequalities: Empirical Analysis

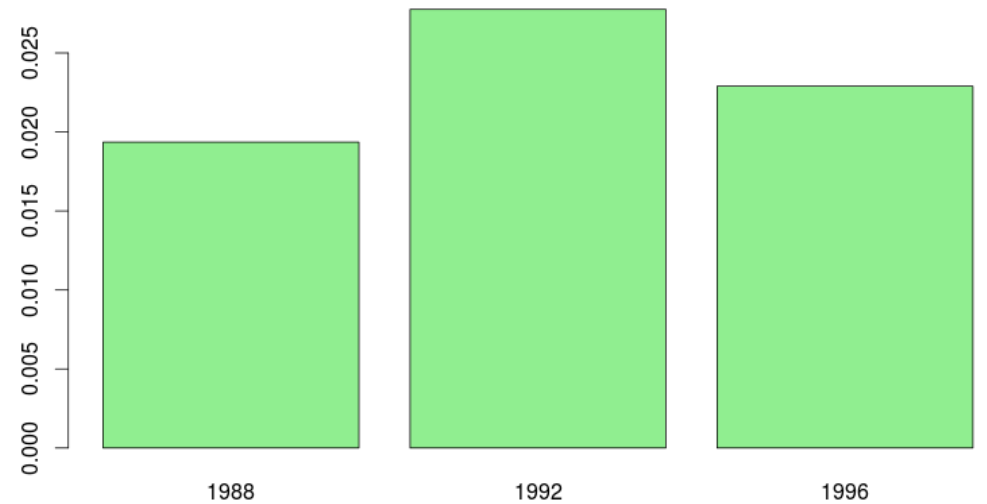
```

1 > P<-poverty(Watts, function(x)
  mean(x)/2)
2 1988... 0.03561864
3 1992... 0.05240638
4 1996... 0.03342492
  
```



```

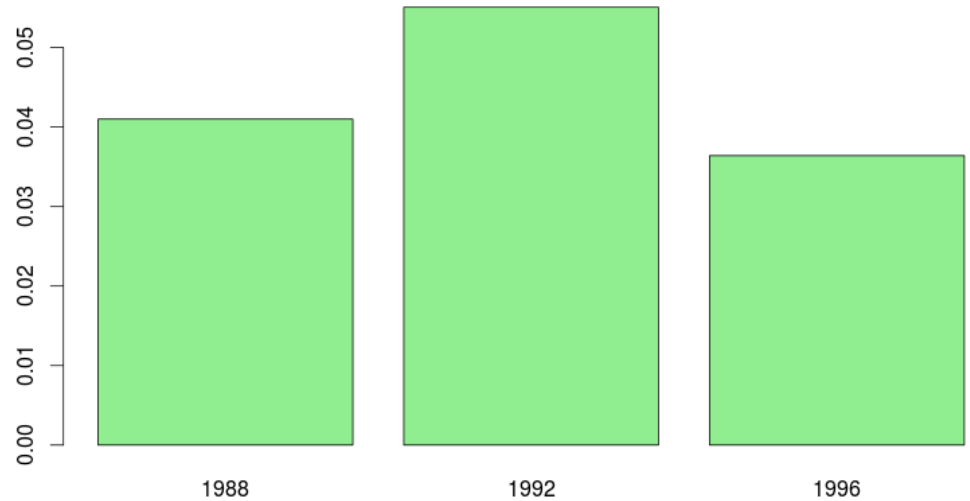
1 > P<-poverty(Watts, function(x)
  quantile(x,.1))
2 1988... 0.01935494
3 1992... 0.0277594
4 1996... 0.02289631
  
```



Inequalities: Empirical Analysis

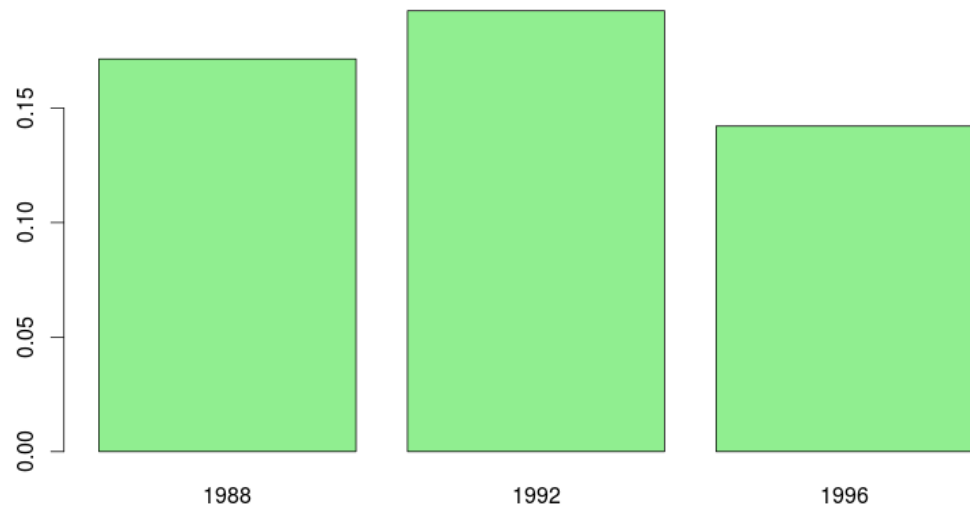
```

1 > P<-poverty (Sen , function (x) mean
  (x) / 2)
2 1988... 0.04100178
3 1992... 0.05507059
4 1996... 0.03640762
  
```



```

1 > P<-poverty (Foster , function (x)
  mean (x) / 2 , param=0)
2 1988... 0.1714684
3 1992... 0.1925117
4 1996... 0.1421479
  
```



Group Decomposability

Assume that x is either x_1 with probability p (e.g. *urban*) or x_2 with probability $1 - p$ (e.g. *rural*). The (total) FGT index can be written

$$P_\alpha = p \cdot \frac{1}{n} \sum_{i,1} \left(1 - \frac{x_i}{z}\right)^\alpha + [1 - p] \cdot \frac{1}{n} \sum_{i,2} \left(1 - \frac{x_i}{z}\right)^\alpha = pP_\alpha^{(1)} + [1 - p]P_\alpha^{(2)}$$

Welfare, Poverty and Inequality

Atkinson (1987, darp.lse.ac.uk) suggested several options,

- neglect poverty, $W(\mathbf{x}) = \bar{x} \cdot [1 - I(\mathbf{x})]$,
- neglect inequality, $W(\mathbf{x}) = \bar{x} \cdot [1 - P(\mathbf{x})]$,
- tradeoff inequality - poverty, $W(\mathbf{x}) = \bar{x} \cdot [1 - I(\mathbf{x}) - P(\mathbf{x})]$,

Projects

- Une réduction de l'inégalité des chances dans l'obtention du revenu en France ? [pdf](#), 2004
- Socioeconomic inequalities in health: Measurement, computation and statistical inference [pdf](#), 1997
- Gini ou Entropie [pdf](#), 2003