

Solutions Exercices Régression linéaire Multiple, Série 1 (suite)

B8

Modèle: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \epsilon_i$

i) $n = 15$

ii) $(X'X)^{-1}$ donnée

iii) $SCE = 282,82$

On cherche à calculer $\hat{\sigma}_{(\hat{\beta}_3 - \hat{\beta}_2)} = \sqrt{\widehat{\text{var}}(\hat{\beta}_3 - \hat{\beta}_2)}$

- $\widehat{\text{var}}(\hat{\beta}_3 - \hat{\beta}_2) = \hat{\sigma}_{(\hat{\beta}_3 - \hat{\beta}_2)}^2 = \hat{\sigma}_{\hat{\beta}_3}^2 + \hat{\sigma}_{\hat{\beta}_2}^2 - 2 \widehat{\text{cov}}(\hat{\beta}_3, \hat{\beta}_2)$

- la matrice de variance-covariance de $\hat{\beta} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{pmatrix}$ est donnée par:

$$\hat{\sigma}_{\epsilon}^2 (X'X)^{-1}$$

où $\hat{\sigma}_{\epsilon}^2 = \frac{SCE}{n-4} = \frac{282,82}{15-4} = 25,71090909$

- $\hat{\sigma}_{\hat{\beta}_3}^2 = \hat{\sigma}_{\epsilon}^2 (X'X)^{-1}_{3,3} = 25,71090909 \times 2,14 = 55,02134545$

- $\hat{\sigma}_{\hat{\beta}_2}^2 = \hat{\sigma}_{\epsilon}^2 (X'X)^{-1}_{2,2} = 25,71090909 \times 0,03 = 0,771327272$

$\Rightarrow \widehat{\text{cov}}(\hat{\beta}_3, \hat{\beta}_2) = \hat{\sigma}_{\epsilon}^2 (X'X)^{-1}_{(2,3)} = 25,71090909 \times 0,11 = 2,8282$

Il vient,

$$\begin{aligned} \hat{\sigma}_{\hat{\beta}_1 - \hat{\beta}_2}^2 &= \sqrt{55,0234545 + 0,771327272 - 2 \times 2,8282} \\ &= 7,080697192 \approx 7,1 \end{aligned}$$

⇒ Réponse C

B9

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 D_i + \varepsilon_i, \quad i=1, \dots, n$$

On donne:

$$\hat{\sigma}_{\beta}^2 (X'X)^{-1} = \begin{bmatrix} 1,89952 & -0,00364 & -0,82744 \\ & 0,00001 & -0,0041 \\ & - & & 2,79655 \end{bmatrix}$$

(Matrice de covariance-variance des β_{reco})

- On cherche : $\hat{\sigma}_{\hat{\beta}_1 + 600\hat{\beta}_2}$

Coe dans **B8**,

$$\begin{aligned} \hat{\sigma}_{\hat{\beta}_1 + 600\hat{\beta}_2}^2 &= \hat{\sigma}_{\hat{\beta}_1}^2 + 600^2 \hat{\sigma}_{\hat{\beta}_2}^2 + 2 \times 600 \text{Cov}(\hat{\beta}_1, \hat{\beta}_2) \\ &= 1,89952 + 600^2 (0,00001) + 2 \times 600 (-0,00364) \\ &= 1,13152 \end{aligned}$$

$$\Rightarrow \hat{\sigma}_{\hat{\beta}_1 + 600\hat{\beta}_2} = \sqrt{1,13152} = 1,063729289$$

⇒ Réponse A $\approx 1,06$

(2)