

PAST CAS AND SoA EXAMINATION QUESTIONS

1. A study was made of the effect of temperature (X) on the yield (Y) of a chemical process. The following data were collected sequentially:

X	1	2	3	4	5	6	7	8	9
Y	1	5	4	7	10	11	12	14	17

You are given the following model:

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \quad i = 1, 2, \dots, 9,$$

where $E(\varepsilon_i) = 0$, $\text{Var}(\varepsilon_i) = \sigma^2$, and $\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$. Calculate the sample autocorrelation coefficient on the Y_i at lag 2. (Answer to the nearest .01.)

- A. .08 B. .25 C. .36 D. .58 E. .62 (83S-3-7)

2. You are given the following series of observations:

t	1	2	3	4	5	6	7	8	9	10
Y_t	1	4	5	7	4	4	8	1	2	4

Calculate $\hat{\rho}_2$, the second sample autocorrelation coefficient.

- A. -.2 B. -.1 C. .2 D. .5 E. .6 (86F-120-11)

3. You are given a chronological sequence of observations Y_t , $t = 1, 2, \dots, 99$. For any value of t , $Y_t = t$. You are given:

$$\sum_{t=1}^{99} t = 4,950 \quad \sum_{t=1}^{99} t^2 = 328,350$$

Calculate $\hat{\rho}_1$, the sample autocorrelation coefficient at lag 1.

- A. .95 B. .96 C. .97 D. .98 E. .99 (87S-120-7)

4. For the random walk model, which of the following are true?

- Over time, the series is stationary about a mean of zero.
- For a given probability level, as the lead time increases, the probability limits for Y_{N+1} approach horizontal bands.
- $\hat{Y}_N(l) = y_N$, $l \geq 1$

- A. 1, 2 B. 1, 3 C. 2, 3 D. 1, 2, 3 E. None of these are correct answers. (87S-120-13)

5. Time series analysis differs from linear regression in that observations are assumed not to be independent. (88S-120-14-MC)

6. The realized values of a time series are 1, 2, 3, ..., 21. You are given:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Determine $\hat{\rho}_2$, the lag 2 autocorrelation coefficient.

- A. .72 B. .79 C. .86 D. .90 E. 1.00 (89F-120-7)

7. A mutual fund has provided investment results for five consecutive years as follows:

Year	1	2	3	4	5
Yield	.07	.06	.07	.10	-.05

Determine $\hat{\rho}_2$, the sample autocorrelation coefficient at lag 2.

- A. -.5 B. -.4 C. -.3 D. -.2 E. -.1 (90F-120-16)

8. According to Pindyck and Rubinfeld, if a data series contains a trend, the trend can be removed by taking successive differences of the data. (91-10-10-MC)
9. According to Pindyck and Rubinfeld, if a data series contains a trend, the trend can be removed by taking successive differences of the data. (92-10-8-MC)
10. Define the stationarity and explain how nonstationary data can be manipulated to create a stationary series. (92-10-34b-1)
11. You observed the following time series:

t	1	2	3	4	5
y_t	1.2	1.1	.9	1.3	1.5

Determine the lag 2 sample autocorrelation ($\hat{\rho}_2$).

- A. -.5 B. -.02 C. .15 D. .22 E. .52 (98S-120-13)

12. You fit an ARIMA(1, 1, 1) model to 110 observations and calculate the sample autocorrelations of the residuals. The sample autocorrelation function for the first nine lags is:

Lag k	1	2	3	4	5	6	7	8	9
$\hat{\rho}_k$.01	-.05	-.11	.13	-.05	.04	.14	-.01	-.02

Calculate the Box-Pierce chi-square statistic (Q).

- A. .1 B. 1.8 C. 3.2 D. 6.1 E. 8.8 (98S-120-17)

13. The graphs below show a time series, its first and second differences, and the sample autocorrelation function for the series and its first and second differences. Based solely on this information, determine which of the following characterizations is most appropriate for the underlying stochastic process.
- A. The process is white noise. *White*
 - B. The process is a stationary process other than white noise. *White*
 - C. The process is a random walk. *White*
 - D. The process is a first-order homogeneous nonstationary process. *White-201*
 - E. The process is a second-order homogeneous nonstationary process. (Sample-4-21) *White-201*

14. The random walk with drift model $y_t = y_{t-1} + d + \varepsilon_t$, where $\text{Var}(\varepsilon_t) = \sigma_\varepsilon^2$, is fit to a time series of 60 observations. You are given:

$$\hat{d} = -2.5 \quad \hat{\sigma}_\varepsilon^2 = 4.0 \quad y_{60} = 57.3$$

Determine the forecast two periods ahead from forecast origin $T = 60$.

- A. 52 B. 55 C. 57 D. 60 E. 62 (98F-120-13)
15. You are given two time series, x_t and y_t . Each time series is assumed to be a random walk. Which of the following statements about these series is correct?
- A. No linear combination of these two time series can be stationary.
 - B. The time series $z_t = x_t - \lambda y_t$ is always stationary for some value λ .
 - C. The time series $z_t = x_t - \lambda y_t$ may be stationary for some value λ that can be determined precisely using regression techniques.
 - D. The time series $z_t = x_t - \lambda y_t$ may be stationary for some value λ that can be estimated by running an ordinary least-squares regression of x_t on y_t .
 - E. None of these answers is correct. (00S-4-28)

16. You are given two random walk models. These models are identical in every respect, except that one includes a known positive drift parameter and the other does not include a drift parameter. Which of the following statements about these random walk models is incorrect?

- A. For the random walk without drift, all forecasted values from time T are equal.
- B. For the random walk without drift, the standard error of the forecast from time T increases as the forecast horizon increases.
- C. For the random walk with drift, the forecasted values from time T will increase linearly as the forecast horizon increases.
- D. For the random walk with drift, the standard error of a forecasted value from time T is equal to the standard error of the corresponding forecasted value for the random walk without drift.
- E. For the random walk with drift, the standard error of the forecast from time T increases or decreases, depending on the drift parameter, as the forecast horizon increases. (00F-4-40)

17. A Dickey-Fuller unit root test was performed on 100 observations of each of three price series by estimating the unrestricted regression

$$Y_t - Y_{t-1} = \alpha + \beta t + (\rho - 1)Y_{t-1}$$

and then the restricted regression

$$Y_t - Y_{t-1} = \alpha$$

You are given:

Price Series	Error Sum of Squares (ESS)	
	Unrestricted	Restricted
I	3,233.8	3,552.2
II	1,131.8	1,300.5
III	211.1	237.0

The critical value at the .10 significance level for the F distribution calculated by Dickey and Fuller is 5.47. For which series do you reject at the .10 significance level the hypothesis of a random walk?

- A. None of these are rejected. B. I, II C. I, III D. II, III E. I, II, III (01S-4-9)

18. Based on 100 observations of a time series you determine the following sample autocorrelation coefficients:

$\frac{k}{\hat{\rho}_k}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	.06	.02	.02	-.16	-.14	-.01	.03	-.01	.01	-.04	.03	.10	-.17	.02	.01

You also determine that the Box-Pierce Q statistic, where $Q = T \sum_{k=1}^{15} \hat{\rho}_k^2 = 9.38$. You must decide if the time series has been generated by a white noise process. Which of the following is true?

- A. If the time series has been generated by a white noise process, then the sample autocorrelation coefficients are distributed approximately according to a normal distribution with mean zero and standard deviation .1.
- B. Because the absolute values of three of the fifteen sample autocorrelation coefficients exceed .1, the probability is 80% that the time series is not generated by a white noise process.
- C. Because none of the absolute values of the fifteen sample autocorrelation coefficients exceeds .2, the probability is 95% that all of the true autocorrelation coefficients are simultaneously zero.
- D. The Q statistic is approximately chi-square distributed with 85 degrees of freedom.
- E. Because the Q statistic does not exceed its critical value at the .05 level of significance, the probability is 95% that the true autocorrelation coefficients are all zero. (01F-4-9)
19. Which of the following is false?
- A. If the characteristics of a stochastic process change over time, then the process is nonstationary.
- B. Representing a nonstationary time series by a simple algebraic model is often difficult.
- C. Differences of a homogeneous nonstationary time series will always be nonstationary.
- D. If a time series is stationary, then its mean, variance and, for any lag k, covariance must also be stationary.
- E. If the autocorrelation function for a time series is zero (or close to zero) for all lags $k > 0$, then no model can provide useful minimum mean square error forecasts of future values other than the mean. (03F-4-17)

20. Which of the following statements about time series is false?
- A. If the characteristics of the underlying stochastic process change over time, then the process is nonstationary.
 - B. A stationary process is one whose joint distribution and conditional distribution functions are both invariant with respect to displacement in time.
 - C. If a time series is stationary, then its mean, variance, and, for any lag k , covariance must also be stationary.
 - D. If the autocorrelation function for a time series is zero (or close to zero) for all lags $k > 0$, then no model can provide useful minimum mean square error forecasts of future values other than the mean.
 - E. A homogeneous nonstationary time series will always be nonstationary, regardless of how many times it is differenced. (04F-4-7)
21. You are given two time series, x_t and y_t , each of which is a random walk. Which of the following is false?
- A. Neither of these two time series is stationary.
 - B. Applying conventional significance tests to the regression of one of these time series against the other will tend to indicate a relationship between the variables when in fact none exists.
 - C. A linear combination of these two time series may be stationary.
 - D. The time series $z_t = x_t - \lambda y_t$ may be stationary for some value λ that can be determined precisely using regression techniques.
 - E. The time series $z_t = x_t - \lambda y_t$ may be stationary for some value λ that can be estimated by ordinary least squares regression of x_t on y_t . (05-ASM-5)

22. You fit an ARMA(1,1) model to 100 observations of a time series. You are given the following residual autocorrelations:

$\frac{k}{\hat{r}_k}$	$\frac{1}{-0.02}$	$\frac{2}{-0.03}$	$\frac{3}{-0.01}$	$\frac{4}{-0.19}$	$\frac{5}{-0.17}$	$\frac{6}{-0.01}$	$\frac{7}{.03}$	$\frac{8}{-0.02}$	$\frac{9}{.02}$	$\frac{10}{-0.04}$
$\frac{k}{\hat{r}_k}$	$\frac{11}{.03}$	$\frac{12}{.12}$	$\frac{13}{-0.17}$	$\frac{14}{.04}$	$\frac{15}{.05}$					

Calculate the Box-Pierce Q statistic based on the first fifteen residual autocorrelations.

- A. 2 B. 4 C. 6 D. 9 E. 12 (05-ASM-8)

23. For a stochastic time series $y_{-1}, y_0, y_1, y_2, \dots, y_T$, you are given:

- i) y_{-1} and y_0 are known.
- ii) $\epsilon_1, \epsilon_2, \dots, \epsilon_T$ are independent random variables with $E(\epsilon_t) = 0$ and $\text{Var}(\epsilon_t) = 1$.

iii) $\Delta^2 y_t = \epsilon_t - .5\epsilon_{t-1}$

operator differential

$\Delta y_t = y_t - y_{t-1}$

Determine the covariance between $y_T - 2y_{T-1} + y_{T-2}$ and $y_{T-3} - 2y_{T-4} + y_{T-5}$.

- A. -1.0 B. -.5 C. 0 D. .5 E. 1.0 (05-ASM-12)