

PAST CAS AND SoA EXAMINATION QUESTIONSA. The Model

- A1. A study was made of the effect of temperature (X) on the yield (Y) of a chemical process. The following data were collected sequentially:

X	1	2	3	4	5	6	7	8	9
Y	1	5	4	7	10	11	12	14	17

You are given the following model:

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \quad i = 1, 2, \dots, 9,$$

where $E(\varepsilon_i) = 0$, $\text{Var}(\varepsilon_i) = \sigma^2$, and $\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$. Calculate the least-square estimate of α . (Answer to the nearest .01.)

- A. -.72 B. -.25 C. 0 D. .25 E. .72 (83S-3-6)

- A2. An economist believes that the three variables P, C, and T are related as follows:

$$PC = kT + \varepsilon,$$

where ε is an error component. The following five observations of P, C, and T are given:

P	1	2	2	1	2
C	2	1	2	3	2
T	3	4	5	6	7

Find the least-squares estimate of k. (Answer to the nearest .01.)

- A. .50 B. .52 C. .59 D. .61 E. .59 (83S-3-8)

- A3. You are given the following model:

$$(Y_t - \mu) = \phi(Y_{t-1} - \mu) + \varepsilon_t$$

You are using this model to represent a time series, for which the first ten values are

t	1	2	3	4	5	6	7	8	9	10
Y_t	8	10	7	6	9	8	6	5	7	4

Conditional on the initial value $Y_1 = 8$ and assuming $\mu = \bar{Y}$, calculate the least-squares estimate of ϕ . (Answer to the nearest .01.)

- A. .29 B. .19 C. .17 D. .05 E. .03 (83S-3-14)

- A4. You are given ten pairs of values (X_i, Y_i) that will be represented by the following model:

$$Y = \alpha + \beta X + \varepsilon,$$

where ε is a random error term with mean 0 and variance σ^2 . You have determined:

$$\sum_{i=1}^{10} X_i = 100 \quad \sum_{i=1}^{10} Y_i = 200 \quad \sum_{i=1}^{10} X_i Y_i = 1000 \quad \sum_{i=1}^{10} X_i^2 = 2,000 \quad \sum_{i=1}^{10} Y_i^2 = 5,000$$

Calculate the least-squares estimate of β . (Answer to the nearest .1.)

- A. 0 B. .1 C. .2 D. .3 E. .4 (84F-3-5)

$$\frac{100 - 10 \times \frac{100}{10} \times \frac{100}{10}}{2000 - 10 \times \left(\frac{100}{10}\right)^2}$$

- A5. A set of observations (X_i, Y_i) has been represented by the following model: $Y = \alpha + \beta X + \epsilon$, where ϵ is a random error term with mean 0 and variance σ^2 . You are given:

$$\bar{X} = 4 \quad \bar{Y} = 2 \quad \hat{\alpha} = 6 \quad \hat{\beta} = 1.5$$

You believe that a better representation of the observed data could be obtained by regressing X on Y , using the following model: $X = \alpha' + \beta' Y + \epsilon'$, where ϵ' is a random error term with mean 0 and variance $(\sigma')^2$. Calculate $\hat{\beta}'$.

- A. .25 B. .38 C. .67 D. .75 E. 1.50 (86F-120-6)
- A6. You are given the following data about a simple regression of the form $Y = \alpha + \beta X + \epsilon$, where ϵ is a random error term with mean 0 and variance σ^2 :

X	1.0	2.0	3.0	4.0	5.0	6.0
Y	10.7	4.9	9.3	14.6	15.7	11.8

Determine the least-squares estimate for the constant term (α).

- A. -3 B. 1 C. 7 D. 11 E. 14 (88S-120-5)

- A7. You are given:

X_i	1	2	3	4	5
Y_i	.25	1.75	1.50	t	2.50

You are representing these data by the following model: $Y = \alpha + \beta X + \epsilon$, where ϵ is normally distributed with mean 0 and variance σ^2 . The least-squares line passes through the point (3, 1.65).

Determine $\sum_{i=1}^5 (Y_i - \bar{Y})^2$.

- A. 2.8 B. 2.9 C. 3.0 D. 3.1 E. 3.2 (88F-120-5)
- A8. Your assistant was to estimate the parameters of a simple regression model of the form:
- $$Y = \alpha + \beta X + \epsilon, \quad X_i = 1, i = 1, \dots, 5$$
- Your assistant determined that the parameter estimates were $\hat{\alpha} = 7$ and $\hat{\beta} = 4$. Later, you learned that your assistant inadvertently found the parameter estimates for the transformed variable $Z = 2Y - 3$. Determine the parameter estimates of the correct regression.
- A. $\hat{\alpha} = 4$ and $\hat{\beta} = 2$ B. $\hat{\alpha} = 4$ and $\hat{\beta} = 8$ C. $\hat{\alpha} = 5$ and $\hat{\beta} = 2$ D. $\hat{\alpha} = 5$ and $\hat{\beta} = 8$
 E. The answer cannot be determined from the information given. (90S-120-8)

- A9. You are performing a simple regression of the form:

$$Y_i = \alpha + \beta X_i + \epsilon$$

i	1	2	3	4	5
X_i	6.8	7.0	7.1	7.2	7.4
Y_i	.8	1.2	.9	.9	1.5

Determine $\hat{\beta}$.

- A. .8 B. .9 C. 1.0 D. 1.1 E. 1.2 (90F-120-6)

A10. Your assistant has estimated the parameters of a simple regression of the form:

$$Y = \alpha + \beta X + \varepsilon$$

as follows: $\hat{\alpha} = 3$ and $\hat{\beta} = 2$.

The data has been partially obliterated:

Observation	1	2	3	4	5	6
X	0.0	2.0	4.0	6.0	8.0	10.0
Y	2.7	7.3	11.5	14.8		

Calculate Y_5 .

- A. 17.7 B. 18.2 C. 18.7 D. 19.2 E. 19.7 (90F-120-8)

A11. You have performed a simple linear regression on 25 observations using the following model: $Y = \alpha + \beta X + \varepsilon$ where ε is a random error term with mean 0 and variance σ^2 . You have determined:

$$\bar{X} = 10.5 \quad \bar{Y} = 31.0 \quad \sum (X_i - \bar{X})^2 = 65 \quad s_Y^2 = 38.4 \quad R = .54 \quad (91S-120-9)$$

Determine $\hat{\alpha}$.

- A. 10 B. 16 C. 24 D. 27 E. 30 (91S-120-9)

A12. You are given two simple linear regression models:

$$Y_i = \alpha_1 + \beta_1 X_i + \varepsilon_{1i} \quad Y_i = \beta_2 X_i + \varepsilon_{2i}$$

If ε_{1i} and ε_{2i} are random error terms with mean 0 and variance σ^2 , $\sum \hat{\varepsilon}_{1i} = 0$. (91S-120-10-MC)

A13. You are given two simple linear regression models:

$$Y_i = \alpha_i + \beta_i X_i + \varepsilon_{1i} \quad Y_i = \beta_2 X_i + \varepsilon_{2i}$$

c'était vraiment faux

If ε_{1i} and ε_{2i} are random error terms with mean 0 and variance σ^2 , $\sum \hat{\varepsilon}_{2i} = 0$. (91S-120-10-MC)

A14. You fit the regression model $Y_t = \beta X_t^2 + \varepsilon_t$ to n observations (X_t, Y_t) . Which of the following is the correct expression for the least-squares estimate of β ?

- A. $\frac{\sum Y_t X_t^2}{\sum X_t^4}$ B. $\frac{(\sum Y_t)(\sum X_t^2)}{\sum X_t^4}$ C. $\frac{\bar{y} \sum X_t^2}{\sum X_t^4}$ D. $\frac{\sum Y_t}{\sum X_t^2}$ E. $\frac{\sum Y_t X_t}{\sum X_t^2}$
(95F-120-1)

A15. You fit the model $Y_i = \alpha + \beta X_i + \varepsilon_i$ to the following data

t	1	2	3
X _i	1	3	4
Y _i	2	Y ₂	5

You determine that $\hat{\alpha} = 5/7$. Calculate Y_2 .

- A. 0 B. 1 C. 2 D. 3 E. 4 (96S-120-1)

A16. You use the method of least squares to fit the model $Y_i = \alpha + \beta X_i^2 + \varepsilon_i$ to the following data:

X_i	0	0	1	2	2
Y_i	2	4	8	16	20

Determine the least-squares estimate of $\hat{\beta}$.

- A. 1.0 B. 2.4 C. 3.7 D. 4.6 E. 5.5 (98S-120-1)

A17. You are given:

i) The model is $Y_i = \beta X_i + \varepsilon_i$, $i = 1, 2, 3$.

ii)

i	X_i	$\text{Var}(\varepsilon_i)$
1	1	1
2	2	9
3	3	16

iii) The ordinary least squares residuals are $\hat{\varepsilon}_i = Y_i - \hat{\beta}X_i$, $i = 1, 2, 3$.

Determine $E(\hat{\varepsilon}_1^2 | X_1, X_2, X_3)$.

- A. 1.0 B. 1.8 C. 2.7 D. 3.7 E. 7.6 (03F-4-29)

A18. Which of the following statements regarding the ordinary least squares fit of the model $Y = \alpha + \beta X + \varepsilon$ is false?

- ok (A) The lower the ratio of the standard error of the regression s to the mean of Y , the more closely the data fit the regression line.
- ok (B) The precision of the slope estimator decreases as the variation of the X 's increases.
- ok (C) The residual variance s^2 is an unbiased as well as consistent estimator of the error variance σ^2 .
- ok (D) If the mean of X is positive, then an overestimate of α is likely to be associated with an underestimate of β .
- ok (E) $\hat{\beta}$ is an unbiased estimator of β . (04F-4-35)

$$\hat{\beta} = \frac{\sum Y_i}{\sum (X_i - \bar{X})^2}$$

A19. You are given:

$$Y_i = \beta + \beta X_i + \varepsilon_i$$

Determine the least-squares estimate of β .

- A. $\frac{\sum Y_i}{\sum X_i}$ B. $\frac{\sum Y_i}{\sum (1 + X_i)}$ C. $\frac{\sum X_i Y_i}{\sum X_i^2}$ D. $\frac{\sum (1 + X_i) Y_i}{\sum (1 + X_i)^2}$ E. $\frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$

(05-ASM-4)

A18: A: plus l'écart type des erreurs est petit, plus les pts sont proches de la droite de régression.