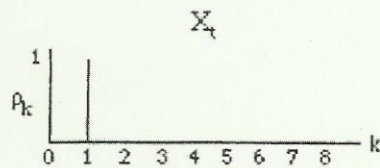


PAST CAS AND SoA EXAMINATION QUESTIONS

A. Moving Average Models

A1. The following graph shows values of the autocorrelation function (ρ_k) for a stochastic process X_t :



X_t could be represented by an MA(1) process. (83S-3-11-MC)

A2. You are using an MA(1) model to represent a time series. You have determined that the lag 1 autocorrelation function (ρ_1) is equal to .4. Calculate θ_1 . (Answer to the nearest .1.)

A. -.5 B. -.4 C. 0 D. .4 E. .5 (84F-3-11)

A3. You are using an MA(1) model to represent a time series of N observations, where N is large. You are given:

$$\mu = 1 \quad \theta_1 = -.5 \quad Y_N = 8$$

Determine $\hat{y}_N(1)$. (Answer to the nearest .5.)

A. 9 B. 9.5 C. 10 D. 11 E. The value cannot be determined from the information given. (85F-3-12)

A4. You are given the following information about an MA(2) model:

$$\theta_1 = .5 \quad \theta_2 = .2$$

Determine ρ_2/ρ_1 .

A. .5 B. .8 C. 1.2 D. 2.0 E. 5.0 (86S-3-15)

A5. The MA(1) model $y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}$ with $|\theta_1| < 1$ is stationary and invertible. (87F-120-14-MC).

A6. Time series analysis differs from linear regression in that residuals from fitting the correct model are expected to be autocorrelated. (88S-120-14-MC)

A7. If the process variable Y follows the first-order moving average model, $y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}$, then the following equation is true:

$$E[(y_t - \mu)(y_{t-1} - \mu)] = -\theta_1 \quad (88F-120-11-MC)$$

A8. If the process variable Y follows the first-order moving average model, $y_t = \mu + \varepsilon_t - \phi_1 \varepsilon_{t-1}$, then random variables separated by more than one time period are uncorrelated. (88F-120-11-MC)

A9. You are given the following MA(2) model:

$$y_t = \varepsilon_t - .2\varepsilon_{t-1} - .7\varepsilon_{t-2}$$

Determine ρ_2 .

A. -.46 B. -.37 C. -.25 D. -.13 E. -.04 (89F-120-1)

- A10. Your statistical consultant analyzed a time series of 50 observations and proposed a model of the form:

$$y_t = \varepsilon_t - \theta\varepsilon_{t-1}$$

The consultant believes that $\theta > .9$. Is the following statement compatible with the proposed model?

$$\frac{\sum (y_t - \bar{y})(y_{t+1} - \bar{y})}{\sum (y_t - \bar{y})^2} = .93 \quad (90S-120-15-MC)$$

- A11. Your statistical consultant analyzed a time series of 50 observations and proposed a model of the form:

$$y_t = \varepsilon_t - \theta\varepsilon_{t-1}$$

The consultant believes that $\theta > .9$. Is the following statement compatible with the proposed model?

$$\frac{\sum (y_t - \bar{y})(y_{t+2} - \bar{y})}{\sum (y_t - \bar{y})^2} = .79 \quad (90S-120-15-MC)$$

- A12. For the MA(1) model, $Y_t = \mu + \varepsilon_t - \theta\varepsilon_{t-1}$, you are given:

$$\frac{E[(Y_t - \mu)(Y_{t+1} - \mu)]}{E[(Y_t - \mu)^2]} = -.4$$

Determine θ so that the model is invertible.

- A. -.7 B. -.4 C. -.1 D. .3 E. .5 (90F-120-5)

- A13. A monthly time series $\{Y_t\}$ has seasonal differences $W_t = Y_t - Y_{t-12}$ that satisfy $W_t = \varepsilon_t - .5\varepsilon_{t-12}$, where the ε_t 's are normally distributed independent random variables with mean 0 and variance 1. Determine the lag 1 autocorrelation coefficient for $\{W_t\}$.

- A. -.5 B. -.4 C. 0 D. .4 E. .5 (91S-120-19)

- A14. For an MA(2) model, you are given:

$$\rho_1 = -.23 \quad \rho_2 = .265 \quad \theta_2 = -.3$$

Determine θ_1 .

- A. -.5 B. -.2 C. .2 D. .3 E. .5 (93S-120-16)

- A15. You are given the following seasonal time series model:

$$Y_t = \varepsilon_t - .4\varepsilon_{t-2} - .5\varepsilon_{t-12} + .2\varepsilon_{t-14}$$

where the ε_t 's are independent, normally distributed random variables with mean 0 and variance σ^2 . Determine ρ^2 .

- A. -.50 B. -.40 C. -.34 D. -.28 E. .40 (93S-120-20)

- A16. You are given a stationary time series model where the autocorrelation function is zero, except at lags 1, 6, 7, and 8. Which of the following yields this pattern of autocorrelations?

A. $y_t = \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-6} - \theta_3\varepsilon_{t-7} - \theta_4\varepsilon_{t-8}$
 B. $y_t = \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-7} - \theta_3\varepsilon_{t-8}$
 C. $y_t = \varepsilon_t - \phi_1y_{t-1} - \phi_2y_{t-6} - \phi_3y_{t-7} - \phi_4y_{t-8}$
 D. $y_t = \varepsilon_t - \phi_1y_{t-1} - \phi_2y_{t-7} - \phi_3y_{t-8}$
 E. $y_t = \varepsilon_t - \phi_1y_{t-1} - \phi_2y_{t-6} - \phi_3y_{t-7}$

(95F-120-16)

- A17. You are given the invertible MA(1) model $y_t = \mu + \varepsilon_t - \theta_1\varepsilon_{t-1}$. You are also given that $\rho_1 = .4$. Determine θ_1 .

A. -.5 B. -.3 C. .1 D. .3 E. .5 (98S-120-14)

- A18. You are given the MA(2) model $Y_t = \varepsilon_t + .8\varepsilon_{t-1} - .5\varepsilon_{t-2}$. Determine ρ_1 .

A. -.7 B. -.6 C. -.4 D. .2 E. .8 (98F-120-14)

- A19. You fit an invertible first-order moving average model to a time series. The lag-one sample autocorrelation coefficient is $-.35$. Determine an initial guess for θ , the moving average parameter.

A. .2 B. .4 C. .6 D. .8 E. 1.0 (00F-4-1)

- A20. You are given the following information about an invertible ARMA time series model:

$$\rho_1 = -.4 \quad \rho_k = 0, k = 2, 3, 4, \dots$$

Determine θ_1 .

A. .2 B. .3 C. .4 D. .5 E. .6 (01S-4-1)

- A21. You are given the following information about a stationary time series model:

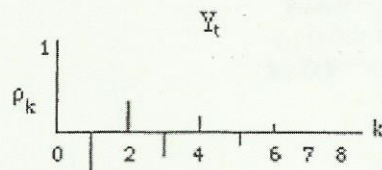
$$\rho_1 = -.310 \quad \rho_2 = -.155 \quad \rho_k = 0, k = 3, 4, 5, \dots$$

You are also given that $\theta_1 + \theta_2 = .7$. Determine θ_1 .

A. .1 B. .2 C. .3 D. .4 E. .5 (04F-4-15)

B. Autoregressive Models

B1. The following graph shows values of the autocorrelation function (ρ_k) for a stochastic process Y_t :



Y_t could be represented by an AR(1) process. (83S-3-11-MC)

B2. The following model is used to represent a time series:

$$y_t - \mu = \phi_1(y_{t-1} - \mu) + \varepsilon_t, \quad t = \dots, -1, 0, 1, \dots$$

Which of the following are true?

1. For this model to represent variables that vary about a fixed level, it is necessary to require that $0 \leq \phi < 1$
2. For random variables following this model, $\rho_k = \phi_k$.
3. For random variables following this model, $\rho_k = -\rho_{-k}$.

A. 1,2 B. 1,3 C. 2,3 D. 1,2,3 E. None of these answers is correct. (83S-3-15)

B3. You are given an AR(2) model with the following autocorrelation coefficients:

$$\rho_1 = .5 \quad \rho_2 = .2$$

Calculate ϕ_1 . (Answer to the nearest .1.)

A. -.5 B. -.1 C. 0 D. .1 E. .5 (84F-3-13)

B4. You are using an AR(2) model to represent a time series of 99 observations, $Y_t, t = 1, 2, \dots, 99$. You are given:

$$\phi_1 = 1/4 \quad \phi_2 = -1/8 \quad \mu = 16 \quad Y_{98} = 14 \quad Y_{99} = 20$$

Determine the expected value of Y_{100} . (Answer to the nearest .01.)

A. 14.75 B. 15.00 C. 17.00 D. 17.25 E. 20.00 (84F-3-15)

B5. You are given that the parameters of an AR(2) model must satisfy the following constraints:

$$\phi_2 + \phi_1 < 1 \quad \phi_2 - \phi_1 < 1 \quad -1 < \phi_2 < 1$$

You are also given that the parameters of an MA(2) model must satisfy:

$$\theta_2 + \theta_1 < 1 \quad \theta_2 - \theta_1 < 1 \quad -1 < \theta_2 < 1$$

Which of the following are true?

1. The parameters of an AR(2) model must satisfy the above constraints to ensure stationarity.
2. The parameters of an MA(2) model must satisfy the above constraints to meet the invertibility condition.
3. If the parameters of an MA(2) model do not satisfy the above constraints, then the process will not necessarily be stationary about the mean.

A. 1,2 B. 1,3 C. 2,3 D. 1,2,3 E. None of these answers are correct. (85F-3-13)

B6. For an AR(1) model, the autocorrelation function (ρ_k) is symmetric about zero. (85F-3-11-MC)

B7. You are given the following information about an AR(2) model:

$$\phi_1 = .3 \quad \rho_1 = .6$$

Determine ρ_2 .

A. .2 B. .4 C. .5 D. .6 E. .7 (86S-3-12)

B8. You are given the following information about an AR(2) model:

$$\rho_1 = .8 \quad \rho_2 = .6$$

Calculate ϕ_1 .

A. .5 B. .7 C. .9 D. 1.1 E. 1.3 (87S-120-15)

B9. You are given the following AR(2) model.

$$\hat{y}_t = .6\hat{y}_{t-1} - .4\hat{y}_{t-2} + \varepsilon_t$$

Calculate ρ_3 .

A. -.3 B. -.1 C. .1 D. .3 E. .5 (87F-120-13)

B10. You are given the following AR(2) model.

$$\hat{y}_t = .5\hat{y}_{t-1} + .4\hat{y}_{t-2} + \varepsilon_t$$

Calculate ρ_3 .

A. .74 B. .76 C. .78 D. .80 E. .82 (88S-120-15)

B11. For an AR(2) model, $\rho_1 = .80$ and $\rho_2 = .82$. Calculate ρ_3 .

A. .728 B. .730 C. .736 D. .742 E. .750 (88F-120-12)

B12. For a first-order autoregressive model, you have determined that:

$$\hat{\phi} = -.6 \quad \hat{\mu} = 50 \quad y_{70} = 46$$

Calculate $\hat{y}(1)$.

A. 46 B. 47.6 C. 50 D. 52.4 E. 54 (89S-120-5)

B13. You are given the following stationary time series model:

$$y_t = \phi_1 y_{t-1} + .5y_{t-2} + \varepsilon_t$$

You are also given that $\rho_1 = .4$. Determine ρ_3 .

A. -.3 B. -.1 C. 0 D. .1 E. .3 (90S-120-12)

- B14. You have fitted an AR(1) model

$$y_t = \delta + \phi y_{t-1} + \varepsilon_t$$

to a time series of length 600. The sample mean, standard deviation and first sample autocorrelation are as follows:

$$\hat{\mu} = 36.00 \quad \bar{s}_Y = .57 \quad \hat{\rho}_1 = .36$$

Determine the appropriate least-squares estimate of δ .

- A. 15 B. 21 C. 23 D. 36 E. 56 (91S-120-14)

- B15. You are given a stationary AR(2) model with the following autocorrelation coefficients:

$$\rho_1 = 2/3 \quad \rho_2 = 0$$

Determine ϕ_2 .

- A. -.8 B. -.5 C. 0 D. .5 E. .8 (91S-120-16)

- B16. The stationary AR(1) model, $y_t = \phi y_{t-1} + \delta + \varepsilon_t$, $t = \dots, -1, 0, 1, \dots$, is fitted to several observed values of a time series. You are given three observed values and two residuals:

$$y_1 = 50 \quad y_2 = 40 \quad y_3 = 38 \quad \hat{\varepsilon}_2 = -4 \quad \hat{\varepsilon}_3 = 2$$

Determine $\hat{\mu}$.

- A. 20 B. 27 C. 37 D. 40 E. 43 (93S-120-14)

- B17. You fit the AR(1) model $y_t - \mu = \phi(y_{t-1} - \mu) + \varepsilon_t$ to the following time series: 14, 5, 11, 4, 11, 3. Determine the Yule-Walker estimate of ϕ .

- A. -.5 B. -.6 C. -.7 D. -.8 E. -.9 (95F-120-19)

- B18. You are given partial autocorrelation coefficients for a stationary stochastic process:

$$a_1 = -.60 \quad a_2 = .36 \quad a_k = 0 \text{ for } k \geq 3$$

Which time series model could have this partial autocorrelation function?

- A. MA(1) B. MA(2) C. AR(1) D. AR(2) E. ARMA(1, 1) (96S-120-15)

- B19. For an AR(2) process with $\mu = 0$, you are given:

$$\rho_1 = .6 \quad \rho_2 = .4$$

Which model most closely corresponds to these autocorrelations?

- A. $y_t - .7y_{t-1} = \varepsilon_t$
 B. $y_t - .6y_{t-1} - .4y_{t-2} = \varepsilon_t$
 C. $y_t - .56y_{t-1} - .06y_{t-2} = \varepsilon_t$
 D. $y_t - .36y_{t-1} - .16y_{t-2} = \varepsilon_t$
 E. $y_t - .09y_{t-1} - .52y_{t-2} = \varepsilon_t$ (98S-120-15)

B20. You are given a stationary AR(p) model with the following autocorrelation coefficients:

$$\rho_1 = .5 \quad \rho_2 = .3$$

Determine the partial autocorrelation at lag 2.

- A. .5 B. .4 C. .3 D. .2 E. .1 (98S-120-16)

B21. You fit the AR(2) model $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$ to a time series of 100 observations. The first two sample autocorrelations are $\hat{\rho}_1 = -.5$ and $\hat{\rho}_2 = 0$. Determine the Yule-Walker estimate of ϕ_2 .

- A. -.50 B. -.33 C. 0 D. .33 E. .50 (98F-120-15)

B22. For the time series y_t , you are given:

t	y_t	$y_t - \bar{y}$
1	984	-16
2	1,023	23
3	965	-35
4	1,040	40
5	988	-12

Estimate the partial autocorrelation function at time displacement $k = 2$.

- A. -.46 B. -.16 C. 0 D. .51 E. .84 (00S-4-20)

B23. You fit an AR(1) model to the following data:

$$y_1 = 2.0 \quad y_2 = -1.7 \quad y_3 = 1.5 \quad y_4 = -2.0 \quad y_5 = 1.5$$

You choose as initial values $\varepsilon_1 = 0$, $\mu = 0$ and $\rho_1 = .5$. Determine the value of the sum of squares function $S = \sum [\varepsilon_t | \varepsilon_1 = 0, \mu = 0, \rho_1 = .5]^2$.

- A. 2 B. 12 C. 15 D. 21 E. 27 (00S-4-39)

B24. You are given the following information about an AR(2) model:

$$\rho_1 = .5 \quad \rho_2 = .4$$

Determine ϕ_1 .

- A. .1 B. .2 C. .3 D. .4 E. .5 (00F-4-25)

B25. For a second-order autoregressive process, you are given:

$$\rho_1 = .53 \quad \rho_2 = -.22$$

Determine ρ_3 .

- A. $< -.70$ B. $\geq -.70$ but $< -.30$ C. $\geq -.30$ but $< .10$ D. $\geq .10$ but $< .50$ E. $\geq .50$
(01F-4-1)

B26. For a stationary AR(2) process, you are given:

$$\rho_1 = .5 \quad \rho_2 = -.2$$

Calculate ϕ_2 .

- A. -.8 B. -.6 C. -.2 D. .6 E. .8 (02F-4-1)

B27. You are given the following information about a stationary AR(2) model:

$$\rho_1 = .5 \quad \rho_2 = .1$$

Determine ϕ_2 .

- A. -.2 B. .1 C. .4 D. .7 E. 1.0 (03F-4-1)

B28. You are given the following information about an AR(1) time-series model:

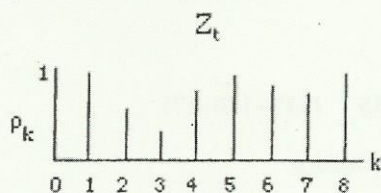
$$\rho_2 = 0.8$$

Determine $|\phi_1|$.

- A. .5 B. .6 C. .7 D. .8 E. .9 (05-ASM-1)

C. Mixed Autoregressive-Moving Average Models

- C1. The following graph shows values of the autocorrelation function (
- ρ_k
-) for a stochastic process
- Z_t
- :



Z_t could be represented by an ARMA (1,1) process. (83S-3-11-MC)

- C2. An ARMA(1, 1) model, which is used to represent a time series (
- y_t
-) has the following parameters:

Mean (μ) = 8 Autoregressive Parameter (ϕ_1) = .5 Moving Average Parameter (θ_1) = -.5

The first three terms of the series are:

$$y_1 = 12 \quad y_2 = 14 \quad y_3 = 10$$

You are given $\varepsilon_1 = 0$. Find the error component (ε_t) at time $t = 3$.

- A. -3 B. -2 C. -1 D. 0 E. 1 (83S-3-12)

- C3. You are given the following ARMA(1, 1) model with mean zero:

$$y_t - .5y_{t-1} = \varepsilon_t - .25\varepsilon_{t-1},$$

where the ε_t 's are independent random error terms with mean 0 and variance σ^2 . Calculate ρ_1 . (Answer to the nearest .1.)

- A. -.1 B. .1 C. .3 D. .5 E. .7 (84F-3-14)

- C4. You are given the following information about an ARMA(1, 1) model:

$$\phi_1 = .8 \quad \rho_1 = .6$$

Determine ρ_2 .

- A. 0 B. .2 C. .3 D. .5 E. .7 (86F-120-13)

- C5. You have fitted the following ARMA(1, 1) model to a time series:

$$y_t = y_{t-1} + \varepsilon_t + .25\varepsilon_{t-1}$$

Based on an origin at 100, you initially forecast $\hat{y}_{100}(1)$ to be 25. Subsequent observation revealed y_{101} to be 28. Determine $\hat{y}_{101}(1)$.

- A. 22.75 B. 24.25 C. 25.75 D. 27.25 E. 28.75 (88F-120-14)

- C6. You are given the following model:

$$y_t = .7y_{t-1} + \varepsilon_t + .5\varepsilon_{t-1}$$

Determine $\sum_{k=1}^{\infty} \rho_k$.

- A. .8 B. 1.5 C. 2.1 D. 2.5 E. 2.8 (90S-120-9)

C7. Given a stationary ARMA(1,1) with the following properties, determine ρ_1 :

$$\rho_2/\rho_1 = .3 \quad \sum_{i=1}^{\infty} \rho_i = .8$$

- A. .17 B. .30 C. .56 D. .62 E. .88 (91S-120-17)

C8. An insurance company uses an ARMA(1,3) model.

$$y_t = \phi_1 y_{t-1} + \delta + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_3 \varepsilon_{t-3}$$

to forecast its claims by calendar quarter. Using many quarters of prior experience, the company has determined:

$$\mu = 100 \quad \phi_1 = .5 \quad \theta_1 = -.2 \quad \theta_3 = .1$$

You are given:

1990 quarter	1	2	3	4
Actual claims	90	98	109	103
Predicted claims	94	100	108	98

Determine the claims predicted by the model for the first quarter of 1991.

- A. 97 B. 99 C. 100 D. 102 E. 103 (91S-120-18)
- C9. According to Pindyck and Rubinfeld, the basic theory behind ARMA models applies to data series with trend. (91-10-3-MC)
- C10. A stationary ARMA(p, q) model is known to have zero partial autocorrelations at all lags $k \geq 2$, and a nonzero autocorrelation at lag one. Determine (p, q).
A. (0, 1) B. (0, 2) C. (1, 0) D. (1, 1) E. (2, 0) (95F-120-15) (Sample-4-25)
- C11. You fit the ARMA(1, 1) model $y_t - \mu = \phi_1(y_{t-1} - \mu) + \varepsilon_t - \theta_1 \varepsilon_{t-1}$ to the following time series:

480 484 482 483 486 484

Determine the sum of squared errors at $\phi_1 = 1.0$ and $\theta_1 = -.1$, assuming $\varepsilon_1 = 0$.

- A. 29 B. 31 C. 33 D. 35 E. 37 (98F-120-16)

C12. You are given the following information about an ARMA(1,1) model:

$$\phi_1 = .3 \quad \theta_1 = .4$$

Calculate ρ_2 .

- A. -.10 B. -.03 C. .03 D. .04 E. .10 (00F-4-17)
- C13. For an ARMA(1,1) model, you are given:

$$y_t = .8y_{t-1} + 3 + \varepsilon_t - .3\varepsilon_{t-1}$$

Calculate ρ_1 .

- A. -.6 B. -.3 C. 0 D. .3 E. .6 (01F-4-39)

C14. Which of the following statements about stationary mixed autoregressive-moving average models is true?

- A. A necessary condition for stationarity is that each parameter ϕ_i must have an absolute value less than 1.
- B. The autocorrelation function approaches ϕ_1 as the displacement increases.
- C. The difference between adjacent forecasted values approaches δ as the number of periods ahead increases.
- D. The forecasted values approach the mean as the number of periods ahead increases.
- E. These models are particularly well-suited to long forecasting horizons. (02F-4-16)

C15. You are given the following time-series model:

$$y_t = .8y_{t-1} + 2 + \varepsilon_t - .5\varepsilon_{t-1}$$

Which of the following statements about this model is false?

- A. $\rho_1 = .4$
- B. $\rho_k < \rho_1, k = 2, 3, 4, \dots$
- C. The model is ARMA(1, 1).
- D. The model is stationary.
- E. The mean (μ) is 2. (03F-4-25)